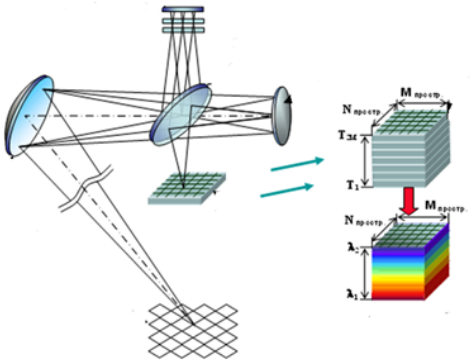
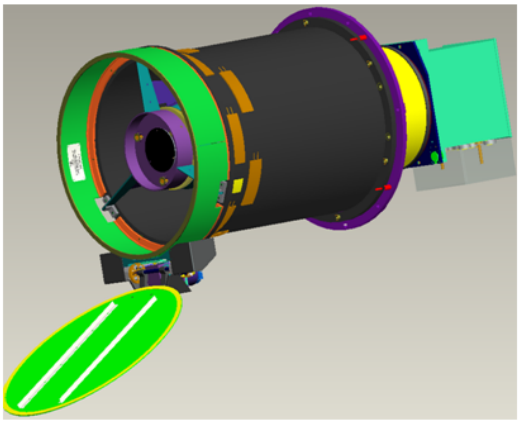
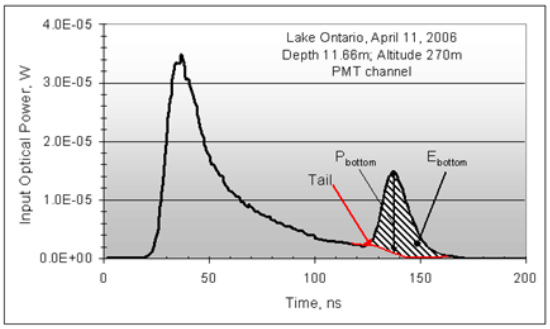
ITMO University

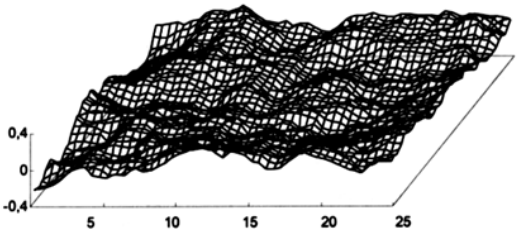
Demin A., Dmitrieva S.

APPLYING MATH MODELLING METHODS FOR FORECASTING THE ENGINEERING SYSTEM STATES









Saint Petersburg

2016

THE MINISTRY OF EDUCATION AND SCIENCE
OF THE RUSSIAN FEDERATION
ITMO University

Demin A., Dmitrieva S.

APPLYING MATH MODELLING METHODS FOR FORECASTING THE ENGINEERING SYSTEM STATES

Educational book

ITMO UNIVERSITY

2016

Demin A., Dmitrieva S. Applying math modelling methods for forecasting the engineering system states. – SPb.: ITMO University, 2016. – 108 p.

The educational book reviews the issues concerning the application of math modelling at forecasting states and behaviour of multi-parameter engineering systems in the context of modern research methods, i.e. according to a new task of mathematical physics – *the task of engineering system synthesis*.

The educational book up in accordance with the State educational standard of higher professional education in the areas 12.04.02 – "OPTICAL ENGINEERING", 09.04.01 – "INFORMATICS AND COMPUTERS" and addressed to the students enrolled in master's degree programs – "optical-digital information-measuring and control systems" and "mathematical models and computer simulation".

Recommended for publication:
– the Department of optical digital systems and technologies (basic Department of the University ITMO) 16.06.2016, Protocol №6;
– Department of Informatics and applied mathematics 21.06.2016, Protocol №5;
– Academic Council of the faculty of computer technology and management ITMO 25.06.2016, Protocol №9.

ITMO UNIVERSITY

ITMO University is a leading higher educational institution (HEI) in computing and photonic technologies in Russia, one of the few Russian HEIs that received the status of a national research university in 2009. ITMO University has been participating in a program aimed at the competitive recovery of Russian universities among the leading world academic centers, known as the “5 into 100” project, since 2013. The ITMO University’s aim is to become a research university of international standard, commercial by its type and focused on the internationalization of all its activities.

CONTENTS

ABBREVIATION LIST

$\mathbb{N}$– set of natural numbers

$\mathbb{R}$– set of real numbers

$\mathbb{R}^+$ set of real positive numbers

$\mathbb{R}^-$ set of real minus numbers

$\mathbb{Q}$– set of rational numbers

$\mathbb{Z}$– set of integers

DWT – discrete wavelet transform

RS – remote sensing

ERS – Earth surface remote sensing

SM – simulation modelling

IP – information process

IS – information system

DM – decision maker

MLM – maximum likelihood method

LSM – least square method

LADM – least absolute deviation method

MPES – multi-parameter engineering system

CWT – continuous wavelet transform

MLE – maximum likelihood estimate

DOS – digital optical systems

DOS_{RS} – digital optical systems for remote sensing

SW – software

ASP – application software package

ES – engineering system

PhD – photodetector

FOREWORD

Scientific achievements of mankind that impress us with its perfection and efficiency are mostly based on results obtained in experimental studies: theorists develop some theories to understand the results of experiments and control the process of its obtaining, then to improve it further. A value of a theory itself is defined by the accuracy of the experiment results description and the ability to interpret them altogether correctly. A new unifying theory in its aggregate development solves the task at first stages of its application, then some new experimental results that cannot be correctly interpreted with the existing theory are accumulated gradually. So the science contradictions rise automatically during the natural evolution of ideas and then the necessity appears to find the contradiction reasons and the ways to resolve it. Present achievements of experimenter physicists have already led to a crash of the physical theories of the XX century: there are so many new results of experiments that the theorists are unable to explain that everyone feels the need to revise both old theories and its fundamentals.

The necessary condition for studying any complex systems is the application of computers at scientific researches, so the conventional approach of association between theories and experiments should be completed with the computer simulation conception. The new effective procedure allows comprehensive studying of the most complicated systems, both natural and created artificially for checking the theory suppositions. Nowadays the computer simulation is applied at almost all the scientific and technical fields, from history to space navigation, as it allows forecasting and simulating the events or estimated phenomena in preselected parameters. For instance, in 1954 E. Fermi, J. Pasta and S. Ulam discovered some surprising peculiarities of atomic behavior in crystals with the computer simulation method thus inspiring active researches of nonlinear systems

and leading to the number of the most important findings in physics and mathematics.

The educational book reviews the issues concerning the application of math modelling at forecasting states and behavior of multi-parameter engineering systems in the context of modern research methods, i.e. according to a new task of mathematical physics – the task of engineering system synthesis. The modern physics requires not just qualitative description of a system and its general principles but an adequate quantitative forecasting, i.e. we need to know physical or chemical properties that could be disregarded before, so we need to create the models that represent physicochemical properties of complex systems fairly in quantity. The modern modelling solves the non-trivial and non-traditional tasks – to complete the fundamental principles with some hypotheses from which the adequate quantitative real-object characteristics could be formulated and applied for synthesis and further for multi-parameter engineering systems forecasting.

The unique educational book "The application of math modelling methods at forecasting the engineering systems states" written by professor A.V. Demin and S.P. Dmitrieva is an example of fruitful approach to this complicated problem. Paying tributes to the developers of the analytical modelling methods, the authors point up their opinion on the unsolved problems and contradictions. It allowed them to establish the fact that it is impossible to find the way of resolving the accumulated contradictions without trying to search for the beginning of formation of the theoretical problems of all modern physics. Most of the scientific theories are similar to mathematics in its internal logic of the derivation because any mathematical theory has several assumptions as its basis, and all the subproducts called theorems are drawn from the assumptions with deductive logical reasoning where the assumptions are some ideal abstract images of real-world objects. In the same way in all exact sciences, the experimental data are accumulated and then the basic laws are formulated from which all the characteristics of various systems and processes embraced by the theory can be

obtained. The solid and precise formulation of science laws are made in mathematical language as some equations. Consequently, some equation or equation system with definite parameter values and definite boundary condition is a mathematical model of any real-world system.

The authors gave a thorough description of the principles they used for argumentation of the forecast model structure, distinguished its base, additional and subsidiary components. The educational book shows computational solutions and forecasting algorithms that confirm the experiment results and thus prove the fruitfulness of the authors' approach in the described research. They connected their research to the functioning of such multi-parameter systems as digital optical systems. The main subjects analyzed with the desired physical simulator are the signal reflection and its passing through the natural and manmade objects describing its *most important elements, i.e.*: *the radiation energy distribution through the length of electromagnetic waves according to the Planck law, Bouguer law describing the light absorption on the propagation path through the scattering or absorbing medium, spectral characteristics of light radiation, reflection and absorption by objects, backgrounds, propagation media and photodetectors, classification of reflection types in which the notion of scattering indicatrix is used, image deformation caused by atmospheric turbulence and image deformation caused by optical system temperature deformation*.

The criteria of the result validation estimation designed by the authors have not only simplified the procedures of searching for the forecast inaccuracy but have given the new procedures for obtaining new science results in key areas of physics, mathematics, informatics, etc. Of course, the new approach to forecasting the behavior of complex engineering systems and its description gives rise to a lot of questions, the answers to which follow from the set of new information from the various chapters of the educational book . To simply the process of searching and learning the information concerning multi-parameter engineering systems, the authors compiled an idioglossary and gave a brief historical background of

forecasting in science thus making the understanding of the main information easier, and, of course, the readers will be grateful to them for such an opportunity.

Every reader having thoroughly studied the educational book will realize that it is impossible to destroy the developed frame of forecasting the multi-parameter engineering systems with math modelling methods. One can only improve and complete it, and the authors have brilliantly done the thing.

Doctor of technical Sciences *A. G. Korobeinikov*

INTRODUCTION

Rationale. The success of human scientific, production and social activities depends much on the information awareness and the opportunities to forecast the states of the activity support tools. The state of environment and engineering systems is characterized by a set of parameters that are probably not connected to each other by a functional relationship, i.e. by a multi-parameter functional. We can define the parameter points real-time but forecasting the future state of environment and engineering systems is a problem of rather a great concern.

The educational book is devoted to the procedures of developing the forecasting models for multi-parameter engineering systems (MPES) in accordance with the aposterior data, i.e. developing the forecasting models according to the current measurement results.

The process of the engineering system forecasting can be summarized by the main aspects and practical tasks described below:

1. Environmental state monitoring;
2. Control of the qualitative and quantitative parameter points of the engineering system functioning;
3. Prevention of the unauthorized tampering of the engineering objects.

The opportunity to forecast the MPES state and behavior is based on the processing of the experimental object characteristics. The basic forecasting principle is the test data meta-extension: by extending some multi-valued statistical cause-and-effect relations and the conclusions drawn from it we may develop some patterns – some laws that tend to display itself not in the proximate values but upon the average, i.e. within some limits of some eventual variance, as a tendency, with a deviation sometimes, thus not allowing to hope for a definite future prediction but making it possible to say that the more consistently some process develops and the tighter the interconnections between the phenomena under investigation are, the more chances we have to obtain a reliable prediction. Within the scope of analyzing the required system,

the task of forecasting the output data basing on the observed input data is called *the observability problem* and arises from the need to predict the future system behavior.

Infra-red light emitted by an object contains the most exhaustive information about it – about the substances it contains, its prehistory and location. Acquiring this information with a detection system and its appropriate processing allow defining and controlling the parameters that are hard or impossible to be measured directly. Electrooptical equipment gathers and analyzes the data concerning the processes taking place in the environment in optical range of the radiation spectrum.

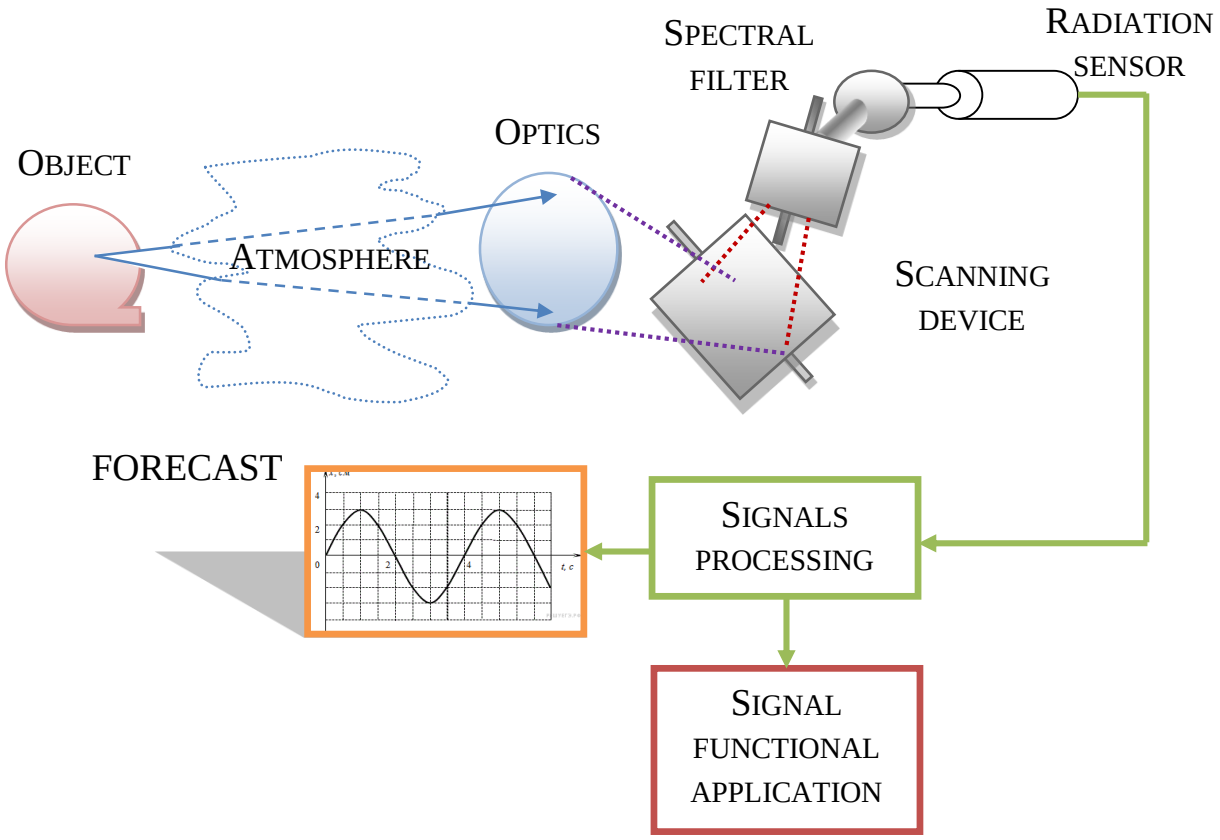


Figure A Radiometric scheme of optical radiation measurement

Solvability of the problems under investigation. To define a problem of exploring the engineering systems and processes, i.e. to set goals, tasks, requirements and limitations for finding a suitable decision, is a key moment for the methods and algorithms of forecast functions development.

A system forecasting function can be developed according to its model representation. Indeed, if $\{M_{OB}(g_i)\}\cap\{OB_{real}(g_i)\}=\max$ the model representation

is the most identical one to the real-world system, and then it is possible to develop a forecast model of its behavior on the following stages:

- stage of changing the real object $\{OB_{real}(g_i)\}$ with the model - $\{M_{OB}(g_i)\}$;
- stage of simulation modelling (SM) $\{M_{OB}(g_i)\}$;
- stage of physical modelling $\{OB_{real}(g_i)\}$.

To perform it we need to set the main properties and peculiar features of the given forecast connection, i.e. the semantics of the described process, where we note all the known physical laws applicable to the present experiment first, and match any limitations and requirements to the physical meaning of variables taking part in functions development, then develop a forecast function describing the given physical process basing on it.

The research paper shows a landmark approach of presenting engineering systems having complex functioning laws with the help of composition and decomposition of its elements with various particularization degrees, and the development of algorithms for implementation of problem-oriented programs for computational experiments. The advantage of applying the given approach is confirmed with the method of evaluating the sufficiency and fidelity of the results obtained.

The fidelity of the scientific results and conclusions is defined by the correctness of applying the mathematical tools and confirmed with the computer simulation results.

The qualitative forecasting questions are of special concern: does the given MPES have a significant growth reserve? Which of them will change which one? What conceptually new technical problems and tasks may arise in future? The existing forecasting methods cannot provide the answers to such questions. The reason lies particularly in the subjectivity of the present methods, i.e. they rely on the estimations, opinions and judgments of the experts. Though the source materials are processed to make it more objective (a formal math model is created, etc.), the most part of it remains subjective in its primitive conjecture, application areas and input information interpretations depending on a scientist's intuition.

Extrapolation is one of the most prospective ES forecasting methods, as it is based on factual objective data in relatively greater degree. The growth curve of some index

characterizing the ES development is constructed on the basis of aposterior information to continue this curve “into the future”; the drawback of the method is that it is not always possible to extrapolate the process development for a short space of time even having the most accurate aposterior data.

The book can be used as a study guide for students and postgraduates getting a degree in the following fields: “Informatics and computer engineering”, “Tool engineering”, “Optical engineering” and probably some other ones.

CHAPTER 1. COMPUTER FORECASTING PROCEDURES

Being a procedure, the forecast modelling does not substitute for mathematics, physics, biology and other sciences, does not compete with them but on the contrary contributes to them as a synthesizing part. Indeed, it is impossible to create a forecast model without relying on the most diverse methods, approaches and innovations – from qualitative analysis of non-linear models to the modern programming languages. The modelling provides additional motivations to various science fields, and adding some, exact knowledge helps limit an instinctive speculative "simulation" extending the application areas for rational methods.

Any theoretical study in reality comes down to the consideration of an object *model*, as a linguistic or mathematical description of any object, process or phenomenon can be made only to a definite particularization level.

A model is some suppositions and analogues representing the real estimative world and having some visualization level or reduced to logical schemes convenient for investigation, simplifying the reasoning and logical constructions or allowing performing the experiments aimed at specifying the character of phenomenon. It is a stand-in of a source object that ensures studying some of the characteristics of a source-object. Figure 1.1 shows the MPES simulation triad.

The modelling is based on the conformity theory stating that the absolute conformity may take place only if one object is changed with the other identical one. We do not achieve absolute conformity at modelling and the scientists want the model to represent the object functioning area under investigation well enough.

The research success depends on the extent to which the model is equal to a real object.

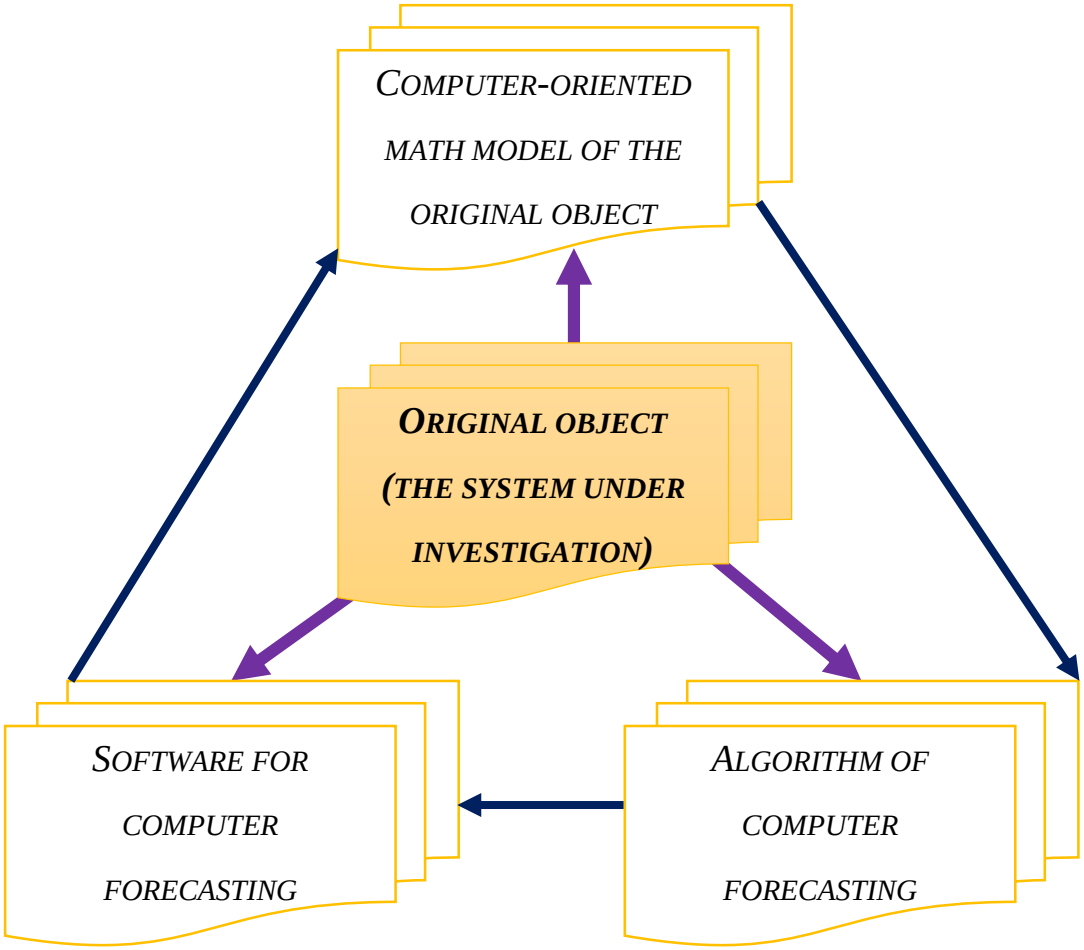


Figure 1.1 The ES simulation triad

Modelling theory is a theory of substituting some objects (source objects) by some other objects (models) and investigating the object characteristics with the help of the models.

While it is being studied, a model acts as an independent quasi-object that allows obtaining some knowledge about the object under research. We develop a forecasting model stage-by-stage; the sequence is shown at Figure 1.3.

The process of developing the best model variation (which as a rule is a compromise one) is rather a complicated one; it implies the comprehensive approach and involves the following stages of model constructing:

Stage one: defining the aim of modelling. Any model is not just an image standing for a source object but its *target representation*.

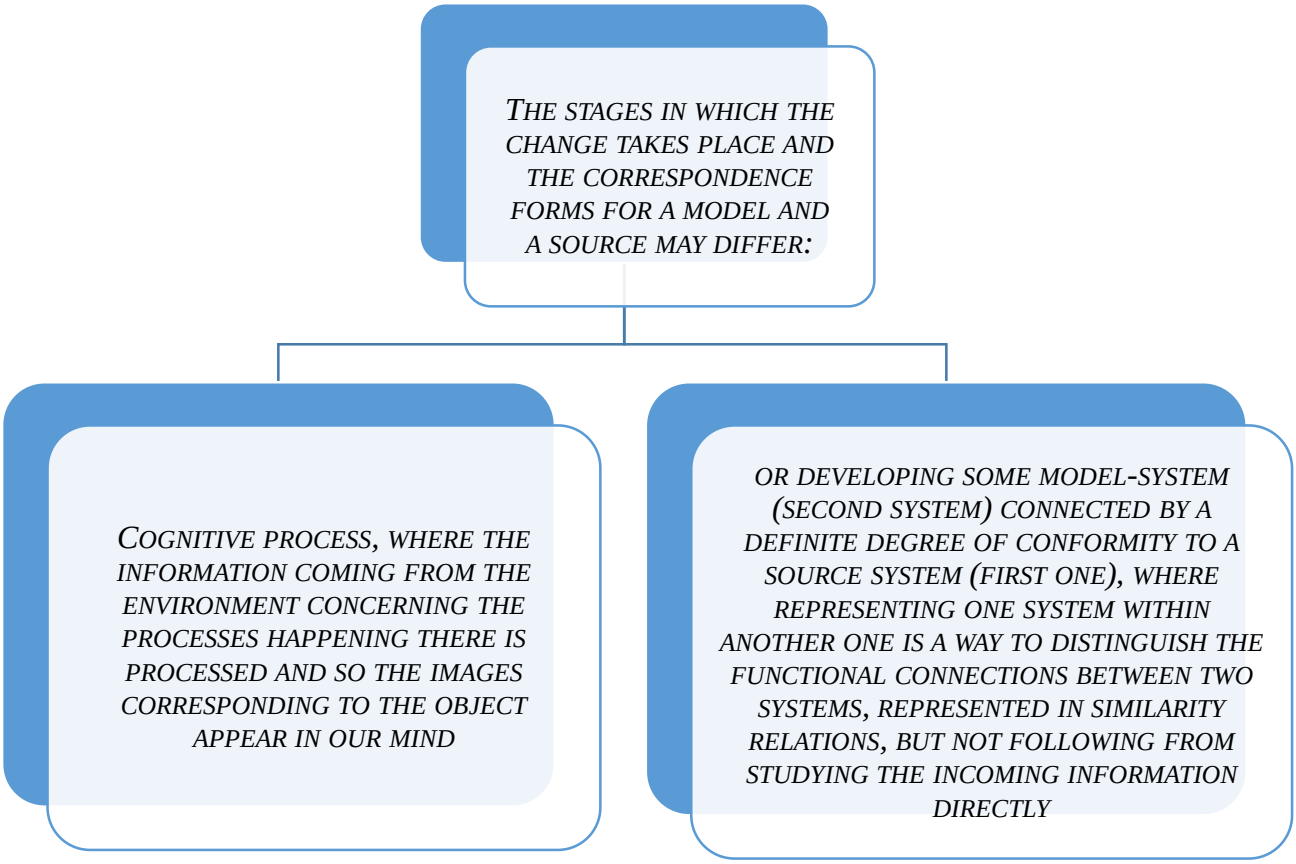


Figure 1.2 Informational processes in modelling

Stage two: model synthesis – developing its possible variations. One should distinguish between:

- *structural synthesis* – developing the model structure: its general form (e.g. the form of system of equations, polynomial or differential ones), number of parameters, etc.;
- *parametric synthesis* – searching for numerical values of model parameters. It is performed either on the basis of the reference data or within the framework of maximum correspondence of the results obtained at the experiment conducted with the model.

Stage three: model analyzing. It means defining the quality of the synthesized model according to the following criteria:

- versatility – the completeness of the object properties represented by the model;

- accuracy – degrees of correspondence of the real data to the ones predicted by the model;
- adequacy – the ability to represent the object characteristics within the framework of modelling aim (i.e. of the task being solved) correctly;
- efficiency – the costs for the model development and implementation.

Stage four. *Choosing and making decision* – evaluating the utility of model variations in general and choosing the best one.

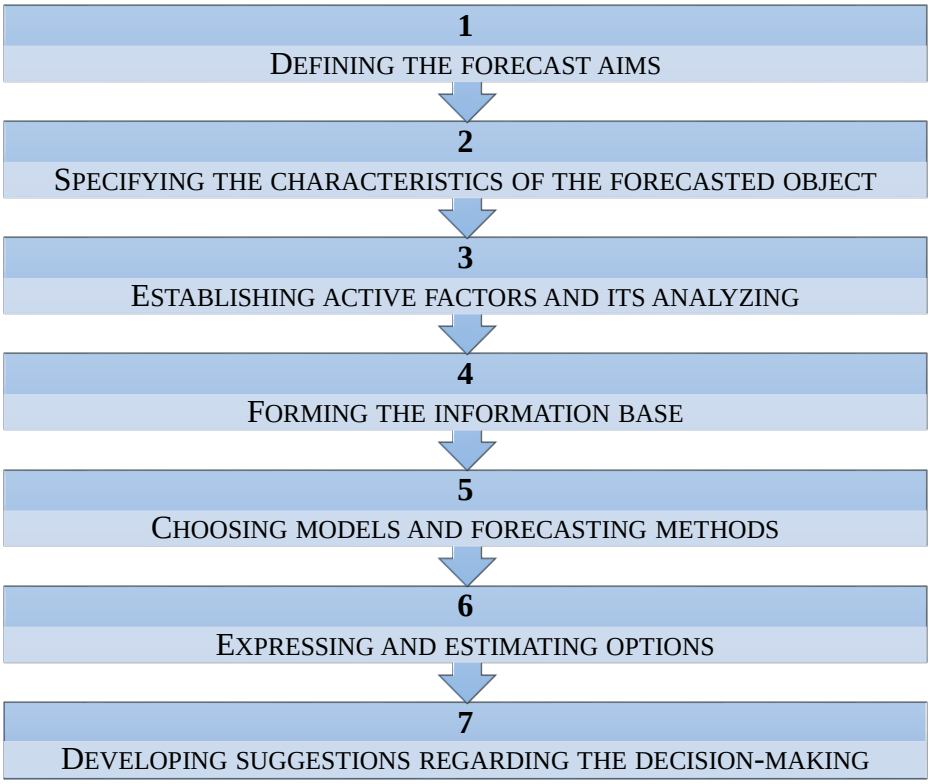


Figure 1.3 Model-developing stages

The problem of highly effective synthesis is closely connected to the problem of choice, i.e. checking to what degree some object or the other one correspond to the given requirements and then accepting or rejecting it.

In the course of solving the given task (modelling aim implementation), the math model undergoes some changes. “Life cycle” term usually refers to the process of some system appearance and development. Here we speak of the *life*

cycle of the model. First stage of the life cycle of the model is arranging the modelling aim that is usually stated verbally (non-formally) into the mathematical language. As a result, we obtain *a description model*. Then we develop *a model of solution* – a set of math expressions providing the way of solving the task. There are three kinds of the model: analytical, computational and simulation ones.

The analytical model is the most accurate on; besides, it allows obtaining a solution in general terms. That is why we should do our best to obtain the analytical model of solution, if possible.

The computational model is a more multi-purpose model; it is highly competitive with the analytical model in accuracy but does not allow obtaining a solution in general terms.

The simulation model is the least accurate but the simplest one. It is applied for obtaining final solutions only in modelling of some complex objects for which we cannot develop the other models of solution. We may apply the simulation model in more simple cases in search of the first approximation for obtaining the final solution with the computational model, or for a preliminary analysis of the object allowing obtaining some initial understanding of the subject of modelling.

The algorithmic model is a solution written in the form of algorithm. It differs from the model of solution, as it is not necessary for the latter one to have all the algorithm characteristics: finiteness, determination, effectiveness, large scale, efficiency. In most cases, the model of solution does not have the property of finiteness.

The program model is an algorithm written on some programming language.

The basis for modelling are *the informational processes (IP)*, because the model development itself is based on the information concerning the real-world object (during the model implementation you receive the information about the given object, at the same time you enter the control data (*the input data*) while performing the experiment with a model, processing the obtained results (*the output data*) is also of great importance.

The computational model development is based on abstracting from the specific nature of phenomena or the source object under study and consists of two stages: developing first the qualitative and then the quantitative model. The more of the notional properties we define and transfer to a computational model, the more real-like it will be and the more opportunities will be provided to a system in which it is applied. The computer-based simulation involves conducting a number of computational experiments aimed at the analysis, interpretation and comparison of the simulation results to the real behavior of the object studied and the subsequent model correction, if necessary.

One can distinguish between analytical and simulation modelling. *At the analytical modelling* one examines the math (abstract) models of a real-world object presented as algebraic, differential or other equations, also involving single-valued computational method leading to its exact solution.

At *the simulation modelling* one examines the math models presented as algorithms representing the functioning of the studied system by sequential execution of a large number of elementary operations at the environmental uncertainty, i.e. while controlling the material flows one should take into account some number of random factors. Under such conditions, it might turn to be impossible or too expensive to develop an analytical model setting the exact quantitative proportions of various components of logistics processes.

The following methods are referred to the *computer-based simulation algorithms*:

- finite-element method;
- method of finite differences;
- method of finite volumes;
- moving cellular automation method;
- method of classical molecular dynamics;
- component chain method;
- node-potential method.

The simulation process is the iterative one and is executed within the limits of the aims formulated before and in line with the simulation borders. The construction starts from studying (investigating) the real-world system, its internal structure, connections between its elements and the external actions and finishes with the model development.

There are four simulation (modelling) stages, from formulating the problem to obtaining results, shown at Figure 1.4.

It is not necessary to move through all of the substages at developing the definite models having definite aims and modelling borders.

The first stage of modelling – *analyzing the requirements and project designing* – includes formulating the conceptual model, developing its formal scheme and checking the efficiency and the expedience of system modelling.

At the stage, we determine the quantitative characteristics (parameters) of the system and its elements functioning, the numerical values of which will be the source data for the modelling. It is obvious, that most of the system parameters are random values. That is why the selection of random variables distribution laws, function fitting, etc. is of great importance at forming the source data. We

need to check the model adequacy as a result of distinguishing the model characteristics and developing the conceptual model. While creating the conceptual model (CM), *the source data area* or *the system informational space* is formed almost simultaneously – it is the stage of source data collecting.

Conceptual model (CM) is an abstract model defining the system body and structure, the element properties and causal relationship of the analyzed system essential for reaching the aims or our modelling. Such models usually describe the nature and parameters (characteristics) of the elementary phenomena of the system under investigation, type and degree of interference between them and the position and meaning of each elementary phenomenon in general process of the system functioning verbally.

At the second stage - *the model development stage* – we specify or choose the modelling software package. At choosing the modelling tools, the software and technical tools are chosen according to a number of criteria. The essential conditions are the sufficiency and completeness of tools for implementing the conceptual model. Usability, being simple and easy to learn, speed and correctness in model creation are among the other criteria.

After choosing modelling environment, the conceptual model formulated at the previous stage is implemented into the computational model, i.e. the model algorithm presentation and particularization is made.

The system model is presented as a set of parts (elements, subsystems). The set includes all the parts ensuring the system integrity maintenance, on the one hand, and the achievement of the preset modelling goals (the required accuracy and fidelity or the results at computational experiments with the model), on the other. Further, we perform the final particularization, localization (distinguishing the system from the environment), structuring (indication and general description of the connections between the selected system elements) and the extended description of behavior pattern of system functioning and probable states.

Setting the simulated time is the next substage. The variable providing the current meaning of the simulated time is called *the simulation clock* at computational model.

There two main approaches to the simulated time advance: from-event-to-event advance and fixed-increment time advance.

Most of the computer programs and most of the engineers developing the models in unified languages apply from-event-to-event time advance.

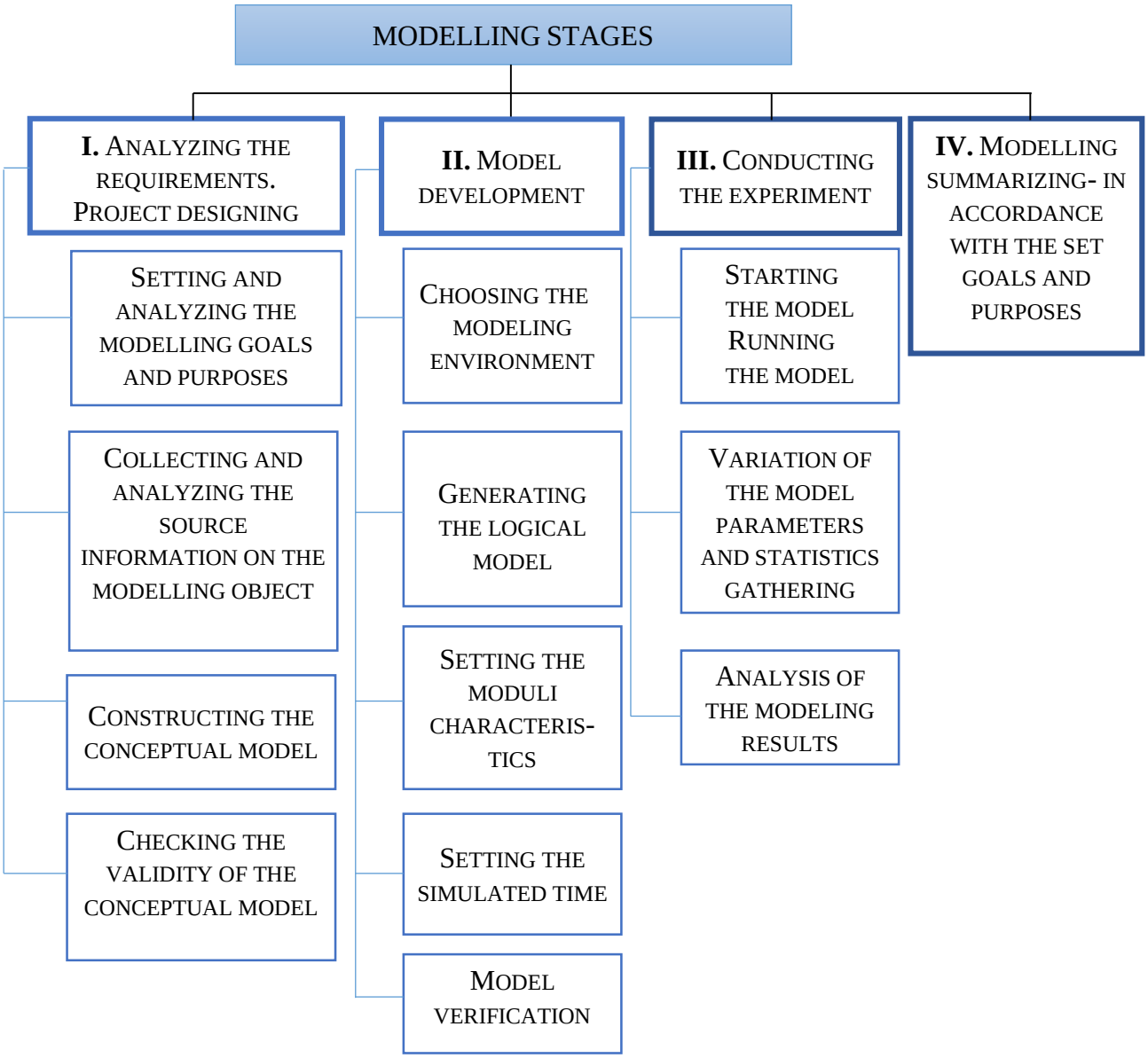


Figure 1.4 Model development scheme

The third stage – *conducting the experiment* – is the critical stage characterized by gathering the required data due to the simulation process and the data static processing at the interpretation of the simulation results according to which the decision is made of whether the research should be continued or finished. If the result is known, we may compare it to the obtained modelling result. The obtained conclusions often promote conducting an additional set of experiments, and sometimes even changing the model. The decision is usually based on the results of experiments and tests. If the results do not correspond to the modelling aims (to a real-world object or process), it means that some mistakes were made at the previous stages or the input data are not the best parameters in the area under investigation, so the engineer returns to one of the previous stages. *Analysis of the modelling results* is the substage at which we make the extensive analysis of the obtained results for receiving some recommendations concerning the system designing or modifying. The stage of *modelling summarizing in accordance with the set goals and purposes* includes the assessment of the work done, the comparison of the set goals to the results obtained and the preparation of the final report on the work done.

On the contrary, interpenetration of all kinds of the modelling and symbiosis of various information technologies in modelling, especially for complicated applications and complex modelling projects, are the principal nowadays trends. For instance, simulation modelling includes conceptual modelling (carried out on the early stages of the simulation model formation), logical-mathematical modelling (including AI methods) – for describing separate model subsystems and also for processing and analyzing the computational experiment results and for decision-making. The procedure of planning and conducting the computational experiment with the corresponding mathematical methods was brought to simulation modelling from physical (full-scale) simulation. Finally, structural-functional modelling is applied at creating the stratified description of many model complexes.

CHAPTER 2 PROBLEMS OF MULTI-PARAMETER ENGINEERING SYSTEMS STATE FORECASTING

The problems of technical process forecasting are rather broad and diverse because there are no strict standards and so a system is developed (synthesized) in the nature of a compromise according to the system purposes, characteristics and limitations see Figure 2.1. The fundamental principles of physics are deep and "elementary" but physics theories come up to great generality and allow describing the attributes of broad classes of physical phenomena. There was a period in the progressive branches of theoretical physics when the analysis of a specific phenomenon was limited to the application of basic principles to a given special case and to a search for the methods of profound conclusions rigorous deduction out of source formulae. The source formulae is not exactly a model at the above described method because some ready-made fundamental axioms and descriptions were used during its development, e.g. a common value of continuous medium assign or a true liquid model.

However, physics has been moving from a traditional task of analyzing the physical world phenomena to *a new task of engineering systems synthesis* during the last 50-60 years, i.e. it is not enough to provide a qualitative system description but an accurate quantitative forecast is required nowadays. Consequently, general principles are not enough, we need to know the object physical or chemical characteristics that may not be taken into account before, i.e. modern physics develops such models that provide the exact quantitative representation of physicochemical properties of complicated systems. Hence, the modelling has to solve the non-trivial and non-traditional task – to complete the fundamental principles with some hypotheses from which the adequate quantitative real-object characteristics could be formulated and applied for complicated MPES synthesizing.

The MPES synthesis is based on developing its *open* or *closed* model and researching it. When choosing and visualizing such a model, one should rely on Bellman principle that states that the art of modelling is the art of going on a narrow track between "a trap of simplifying too much" and "a morass of complicating too much". That is why a model should be complicated enough to include all the necessary components, and simple enough "to be able to see the woods for the trees". One should not work for a very complicated model. *N*. Wiener said specially for complicated model devotees: "Only a cat can be a final model of a cat". A model should be easy to remember and accessible for application not only for inventors (at *heuristic synthesis*) but for computing engineers also (at *statistical synthesis*).

The systems theory that customary divides the design into two large sections (external and internal design) is taken into account at MPES designing, see Figure 2.2.

Since MPES is *an information model*, it is a dual-purpose model and should be synthesized (simulated) with the involvement of physics and information theories. According to the system analysis and synthesis methods peculiar to these two theories, we may consider two classes of system models shown at Figure 2.3 [47]: physical and information ones.

A dominant information approach [47.*p*.94] *is a key to developing a correct information model. According to it, instead of applying the old Hartley and Shannon concept of the required exclusion of psychological factors or N. Wiener's hints that the information is neither substance nor energy, information should be treated as an intermediary between spirit and substance, having three components: purpose, quality, quantity.*

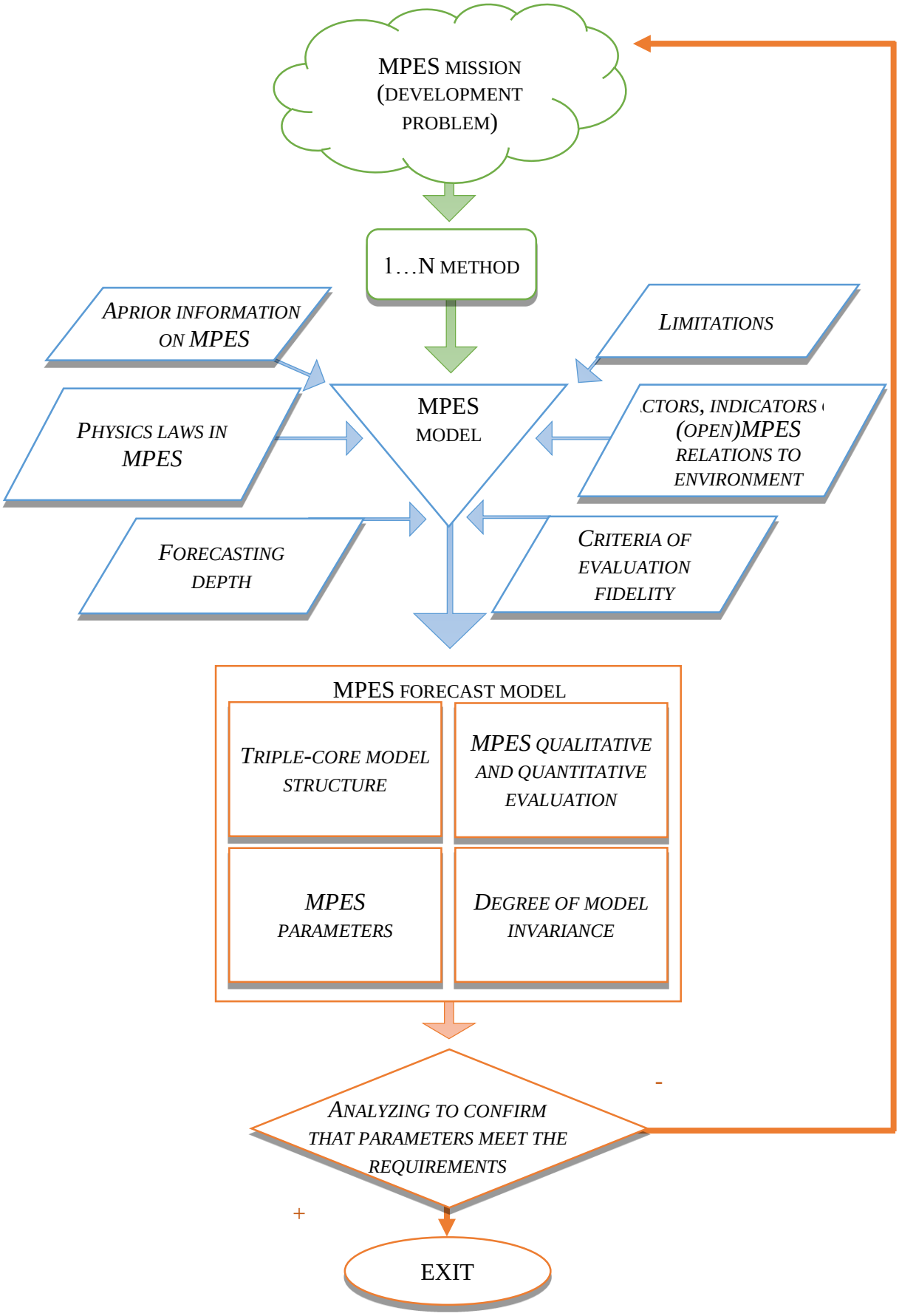


Figure 2.1 Probable structure of MPES forecast model synthesis

The main issues investigated with MPES physical models in the context of digital optical systems are the signal reflection from or passing through nature and manmade objects, see Figure 2.4. Let us describe its most important components:

- radiation energy distribution through the length of electromagnetic waves according to the planck law;
- bouguer law describing the light absorption *on the propagation path through the scattering or absorbing medium*;
- *spectral characteristics of light radiation, reflection and absorption by objects, backgrounds, propagation media and photodetectors*;
- *classification of reflection types (mirror, diffuse or mixed reflection) in which the notion of scattering indicatrix is used*;
- *image deformation caused by atmospheric turbulence*;
- *image deformation caused by optical system temperature deformation*.

Developing a forecast model of behavior for a one-parameter, ES does not usually cause any problems. However, for multi-parameter engineering systems that we label as $\{OB_{real}(g_i)\}$ and that operate off-line open-loop, it is quite challenging to develop a forecast model of its state, labelled as $\{M_{OB}(g_i)\}$, and especially to create an algorithm for such a development.

Further we consider the algorithm of a forecast model development for $\{OB_{real}(g_i)\}$ on the example of an airborne system of the Earth surface optical range remote sensing (Figures 2.4, 2.5, 2.6), the functioning of which is based on the following principles: *optical radiation propagation principle, laws of motion, principles of optical data processing, control engineering, computer engineering* $\{OB_{real}(g_i)\}$.

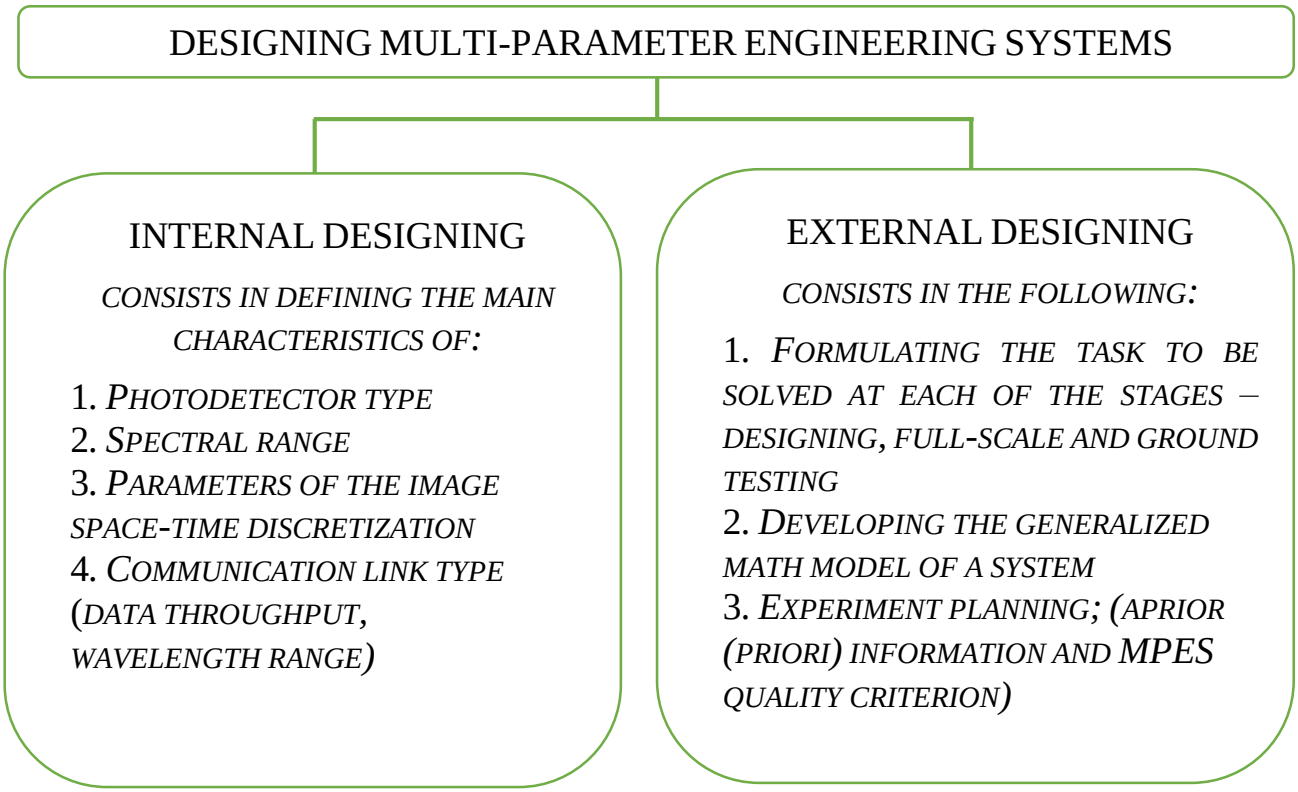


Figure 2.2 External and internal MPES designing

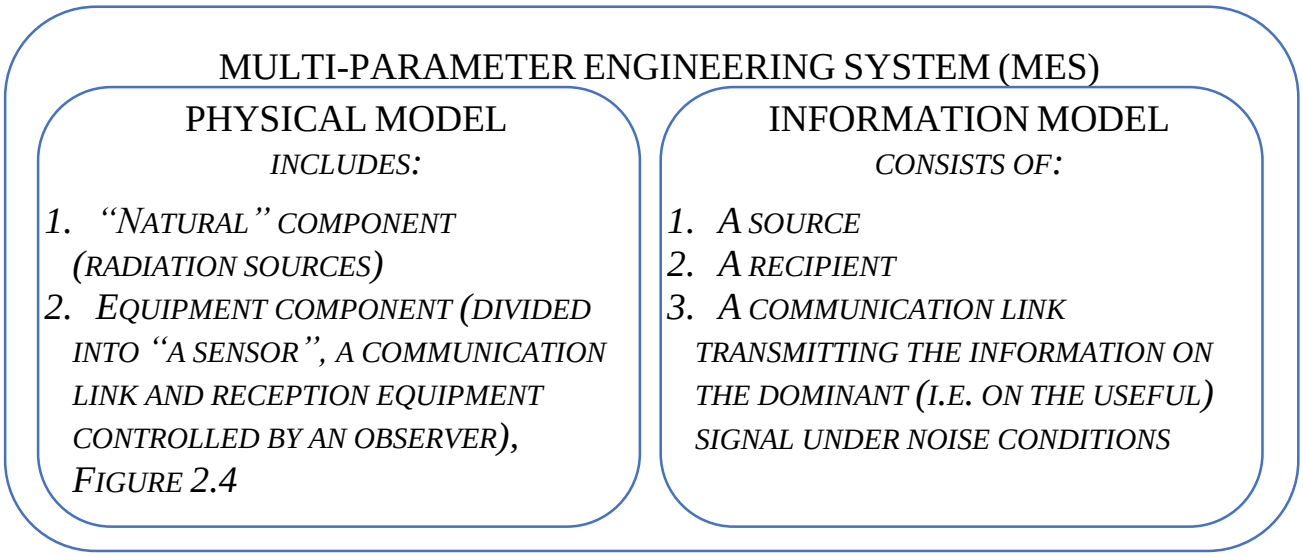


Figure 2.3 MPES – synthesis of information and physical models

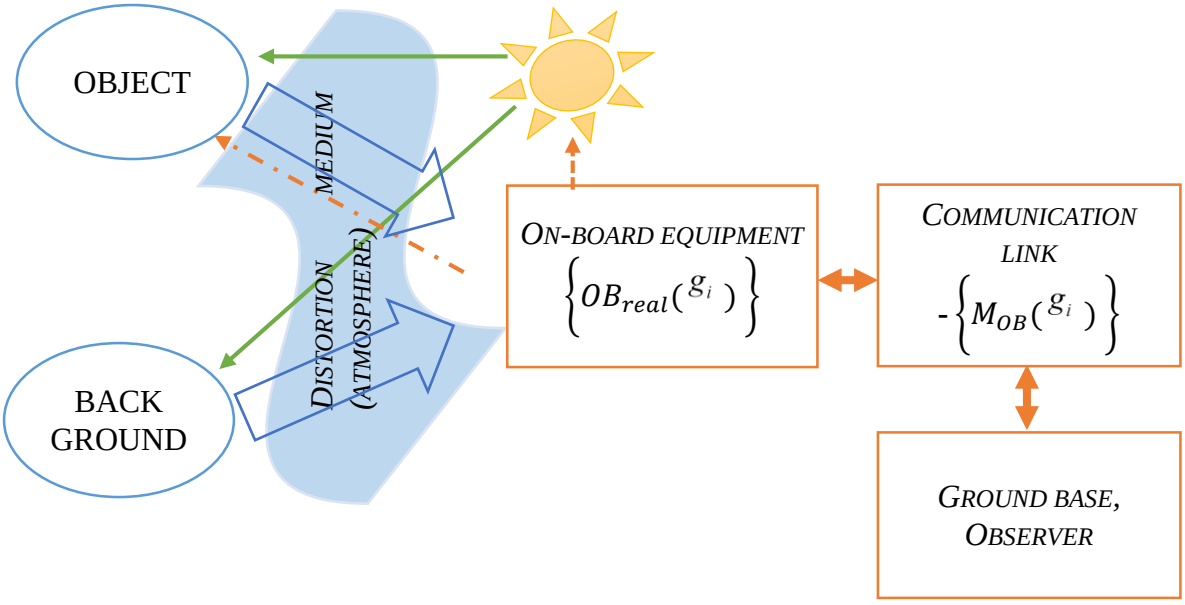


Figure 2.4 Generalized physical model of an on-board multi-parameter digital optical system; two connections of a useful object and background to the on-board equipment (solid arrows) mean the existence of self-radiation and reflected radiation; the connection of the on-board equipment to an exposure source (dot line) means the existence of systems of two types – with natural and artificial lighting; backward link of the on-board equipment to an object (dot-and-dash line) represent the process of choosing an object during the observation.

When applied, to develop a system forecast model, it is required to compare its mathematical structure to the structure being known at the moment of carrying the research out:

$$\mathrm{M}_{str}^{(\mathrm{T})} = \ll S_1, S_2, \ldots, S_k; R^{(1)}, R^{(2)}, \ldots, R^{(n)}; P^{(1)}, P^{(2)}, \ldots, P^{(m)} \gg . \qquad (2.1)$$

where $\{S_i\}_1^k$ is a set of math members different in purposes, names and function operations; $\mathrm{R}^{(\mathrm{n})}$ is a subset of n-th power product; $\{P^{(j)}\}_1^m$ are the mapping operators.

For instance, a mathematical structure of a modulation transfer function of the Earth surface remote sensing system MTF_{ERS} is a product of MTF components (the Earth surface; the atmosphere; the digital optical system; the system of

information reception and conversion), each of them is a function of different structure, precisely:

$$\begin{cases} MTF_{ERS} = MTF_{ES} \times MTF_{Atm} \times MTF_{DOS} \times MTF_{SIRC} \\ MTF_{Atm}\left(\nu_y, \nu_z\right) = \exp\left\{-2\pi^2 \dfrac{Q_T^2 \cdot f^2 \cdot \left(\nu_y^2 + \nu_z^2\right)}{H_{cam}^2}\right\} \end{cases} \quad (2.2)$$

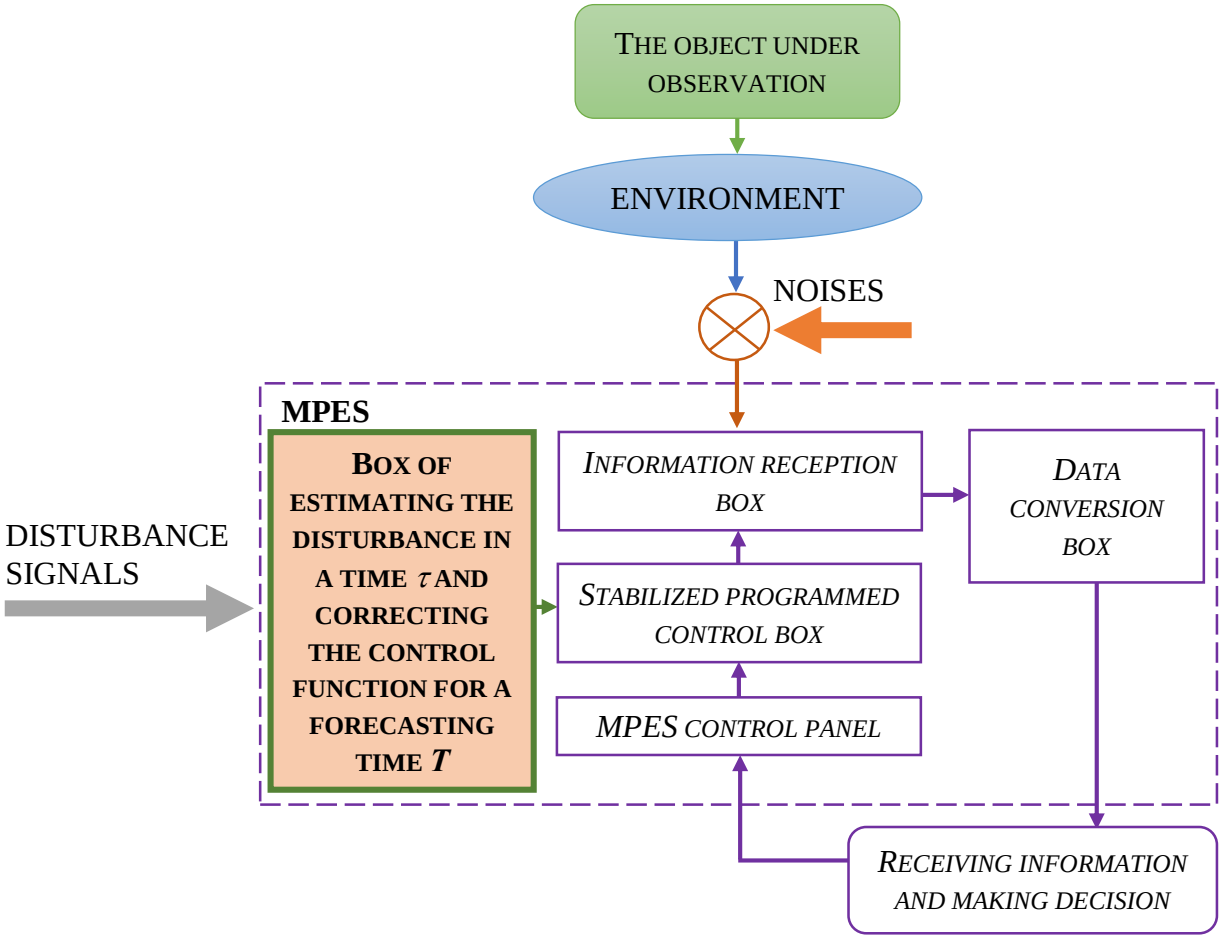


Figure 2.5 Diagram for closed-loop multi-parameter engineering system having forecast control

The following symbols are used for MTF_{Atm} in formula (2.2): ν_y, ν_z - space frequencies; Q_T - atmospheric turbulence parameter changes from 0.017 to 0.1; f - lens focal length of ERS system; H_{cam} - the height of the shooting, without breaking the reasoning integration the equations for MTF_{ES}, MTF_{DOS} and MTF_{SIRC} are not formulated. We can see from formula (2.2) that a real

MTF_{ERS} value based on the ground-based researches is a forecast value, if just for Q_T and H_{cam}. That is why the results of researches are of great importance for developing a forecast model.

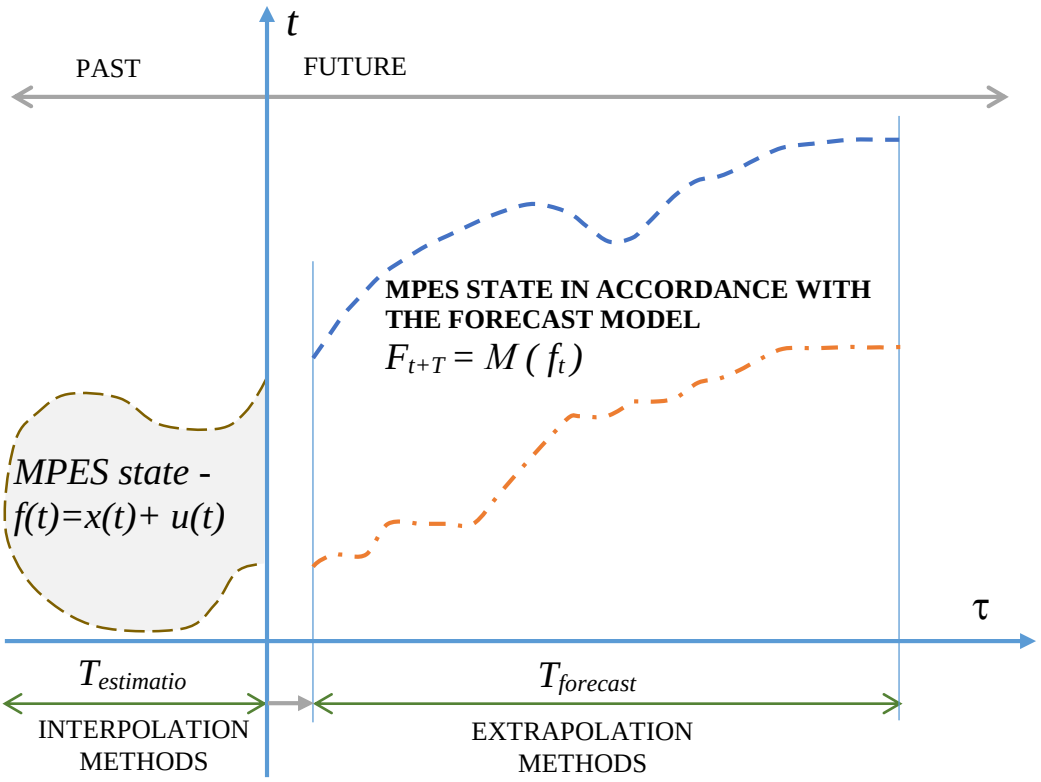


Figure 2.6 Diagram of a probable multi-parameter engineering system forecasting basing on aprior data

For developing the MPES forecast model we suggest considering processes on account of the opportunity of dividing the general control task into fractions or decompositions, where its own model part may be developed and applied for each of decompositions, assuming the existence of a "complete" model and describing mutual influence of its parts on condition that they have maximum freedom from the would-be methods of solving the given fractions, at the same time basing on its content and dynamic properties of the investigated technical process. We pay special attention to the fact that in case there being some mutual influence in the required system of MPES models, there should exist some model having the dynamic description of the influencing processes in it. For instance, for describing

the flying vehicle drive we often use a driving force model presented as statistical dependence on several motion parameters, and that is quite enough for controlling the vehicle in the routine flight modes but at the violent maneuvering we need to know the dynamics of the driving force changing. Consequently, there can even be no “complete” model at the development of the MPES control algorithm because it is replaced with a system of decomposition tasks, i.e. it is expected that there exists a system of “windows” in a “black box” through which the required processes can be observed.

CHAPTER 3. ANALYTICAL REPRESENTATION OF EXPERIMENT RESULTS

3.1 Test data analysis

The source test data $\{x_i\}_1^n$ may be specified either in tabular or graphical form. At graphical form, the data may be represented step-wise in analytical form of $F\left(\{x_i\}_1^n\right)$, in particular [5, p.328]:

1) At the first stage we need:

- reduce the data dimension by applying *the principal component analysis* (*PCA*), it should be noted that the volume of data loss is as small as possible here;
- define the possible characteristics and distinctive features of the required analytical dependence $F\left(\{x_i\}_1^n\right)$, i.e. to define its acceptable region.

2) At the second stage, depending on the $F\left(\{x_i\}_1^n\right)$ acceptable region, we need to do the following:

- if we search for $F\left(\{x_i\}_1^n\right)$ only for analytical representation of the experiment results, i.e. we are not going to extend its "application" area, it is reasonable to use the interpolation method with subsequent approximation;
- if we search for $F\left(\{x_i\}_1^n\right)$ with regard to the extension of its application beyond the range of $\{x_i\}_1^n$, then it is reasonable to use the extrapolation method with subsequent approximation.

It should be noted that if source data are given in graphical form, we should transform it to a discrete form, i.e. to digitize it. Sample spacing size (Δx) and the upper frequency in the analogue signal spectrum ($\nu_{\max}$) are connected according to the sampling theorem: $2\Delta x \cdot \nu_{\max} = 1$.

If the frequency and the source data rangeability are high (volatility level is high), we apply various processing methods, the main of which are given in Table 3.1 [10, 34, 52, 54, 58].

Data processing methods

Table 3.1

METHOD	MATHEMATICAL METHOD REPRESENTATION
Normalization method	$\widetilde{x}_i = \frac{x_i - \min\left(\{x_i\}_1^n\right)}{\max\left(\{x_i\}_1^n\right) - \min\left(\{x_i\}_1^n\right)}$
Arithmetic mean method	$\overline{X} = \frac{\sum_1^n x_i}{n}$
Arithmetic weighted mean method	$\overline{X} = \frac{\sum_1^n x_i w_i}{\sum_1^n w_i}$
Uniform approximation method (min-max method)	$\left\|f(x) - g(x)\right\| \prec \varepsilon; x \in [a,b]$
Conditional-relative differences method	$\sum_0^n \frac{(y_R - y_{zad})^2}{y_{zad}^2} \to \min$
Exponential smoothing method	$\frac{\exp\{-t/\tau\}}{\tau}$; $\sim 1{,}5T_{Набл}$..
N-fold exponential smoothing method	$\frac{t^{n-1}\exp\{-t/\tau\}}{\tau^n(n-1)!}$ $\left(aT_{Набл}/2n\right)$, if n>3, then $(a/2n)$<1
Alternating smoothing method	$\begin{cases} \nu^{-1}\left(\nu^2 + \tau^{-2}\right)\cdot(\sin \nu t)\cdot\exp\{-t/\tau\} \\ T_{Набл} = 3\tau + 0{,}5\tau\cdot\ln\left[1 + (\nu\tau)^{-2}\right] \end{cases}$
Parabolic smoothing method	$\frac{6\tau(t_y - \tau)(kt - \tau^2)}{t_y^3} ... \tau \in [0; t_y]$

At source data processing we define some special and irregularly different x_i that may turn out to be both outliers of the series [46, p.143] and significant extreme points relating to data semantics. Both the $F\left(\{x_i\}_1^n\right)$ value "violation" of acceptable region

specified physical nature of a phenomenon at some points $\{x_i\}_1^n$, and the excess of some threshold value δ between two neighbor reports, i.e. $|F(\{x_{i-1}\}) - F(\{x_i\})| \succ \delta$ or $|F(\{x_i\}) - F(\{x_{i+1}\})| \succ \delta$, may serve as an outlier test. If possible, they should be deleted before data investigating – at the stage of primary data processing. However, *sometimes an outlier provides such kind of information that cannot be obtained from the other data because it is connected to an unusual combination of conditions that is of vital importance.* Nevertheless, we should exclude the outliers instantly if we find out that they are caused by such reasons as the errors in logging, in equipment alignment [23, p.199] or by applying non-certified equipment.

3.2 Methods of analytical form representation of experiment results

Having completed the data investigation, we have to choose a method of developing the analytical dependency and to work out the criterion of defining the match of the obtained analytical model to a source test data. Table 3.2 describes the most frequently used development methods.

Methods of data analytical representation

Table 3.2

METHOD	MATHEMATICAL METHOD REPRESENTATION
Least square method (LSM)	$\begin{cases} F(a,b) = \sum_1^n [y_i - (ax_i - b)]^2 \\ \frac{\partial F(a,b)}{\partial a} = 0; \frac{\partial F(a,b)}{\partial b} = 0 \end{cases}$
Moment method	$\hat{\alpha} = \frac{\pi}{\sqrt{6}} \cdot \frac{1}{\sqrt{\hat{\mu}_{2x}}}, \hat{\beta} = exp(-\hat{\alpha}\hat{m}_{lz} - C)$

Maximum likelihood method (MLM)	$L(\theta) = w(\bar{y}\ /\theta\ ) = \prod_{i=1}^{n}(y_i\ /\theta\ ) = = \frac{1}{\left(\sqrt{2\pi}\sigma\right)^n} exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - S_0 G(\theta, \theta_i))^2\right)$
Wavelet method	$W_\Psi(a,b)f(x) = \frac{\int \frac{1}{\sqrt{\lvert a\rvert}}\Psi\left(\frac{x-b}{a}\right)f(x)dx}{\sqrt{C_\Psi}}$
Spline function method	$S_m(x) = \bigcup_{0}^{n-1} S_{m,i}(x)$
Least absolute deviation method	$\lVert Y - f(x)\rVert = \sum_{1}^{n}\lvert y_i - f(x_i)\rvert$
Mean-value method	$\sum_{i=1}^{n}(y_i - \bar{y}) = 0$

Maximum likelihood method (MLM) is one of the most methods of estimating the model (both linear and non-linear) parameters and highly effective Bayesian estimation method at a simple loss function.

The "MLM" notion appeared in Fisher paper in 1922. The method provides the estimation of *constant parameters at uniform prior distribution* and is the basic method if the prior distribution is not specified, i.e. when the *estimation of an unknown parameter* θ *by observations* $y_1, y_2 \ldots y_n$ is meant. According to the method, the estimates of the unknown model parameters are found on condition of likelihood function maximization [40, p.237].

The likelihood function L is the joint sample distribution, which is the function of parameter

$$\theta = (\theta_1, \theta_2, \ldots \theta_k), \qquad (3.1)$$

where $(\theta_1, \theta_2, \ldots \theta_k)$ is the vector of unknown model parameters.

If the sample has continuous distribution, the likelihood function L is described with the joint distribution density

$$f(x, \theta) = f_\theta(x_1) \cdot f_\theta(x_2) \cdot \ldots \cdot f_\theta(x_n). \quad (3.2)$$

If the sample elements $X = (X_1, X_2, \ldots, X_n)$ have discrete distribution, then the likelihood function L becomes the form:

$$f(x, \theta) = P_\theta(X_1 = x_1) \cdot f_\theta(X_2 = x_1) \cdot \ldots \cdot P_\theta(X_n = x_n). \quad (3.3)$$

Value $L(\theta)$ may be considered as *a measure of value θ likelihood at a specified implementation x.*

Let us suppose that L is a sample likelihood function and at the observed values is the function of parameters θ : $\theta = (\theta_1, \theta_2, \ldots \theta_k)$, then the most reasonable values of $\hat{\theta}$ that maximize the function L are called *the values of maximum likelihood of θ*:

$$\hat{\theta} = argmax_\theta L(x, \theta). \quad (3.4)$$

It is evident that the estimates $x_1, x_2, \ldots, x_n$ depend on the observations and are highly effective in sweeping assumptions. It often happens that it is easier to search for the function maximum point $ln\ L(\theta)$, that aligns with $\hat{\theta}$ due to the log monotonicity, where θ is the element of space Ω. If Ω is an open interval and $L(\theta)$ can be differentiated and reaches a maximum at Ω, then the maximum likelihood estimates satisfy the following equation:

$$\frac{dL(\theta)}{d} = 0\,. \quad (3.5)$$

Maximum likelihood conception may be considered as a main idea of all the methods on which the test data statistical processing for developing the empirical equation is founded. In general, we may formulate the maximum likelihood conception as follows: the best phenomenon description is the one that provides maximum probability to obtain exactly that values that in fact were obtained. [33, p.104].

Maximum likelihood method and function Bayesian estimation will be considered in detail in Chapter 5.

At normally distributed errors ε_i, the maximum likelihood method is reduced to the least square method (LSM) [25, p.237-238] that consists in minimizing the sum of differences squares between the source testing and corresponding values of function derived from the approximation (3.6) [39, p.42-43]:

$$\sum_{i=1}^{n}\left(y_{rasch_i} - y_{zad_i}\right)^2 \to min, \qquad (3.6)$$

where y_{rasch_i}- is the desired value, y_{zad_i} – the value specified by the source test data.

The least square method (LSM) is one of the most wide-spread approaches to solving the formulated task nowadays. Its main peculiarity is the absence of any strict requirements to the prior information concerning the evaluated parameters and experimental errors [47, p.44].

As we have already noted, the LSM usually suggests that the errors are governed by the normal law of distribution. However, a number of researches prove the good forecasting properties of models derived in accordance with LSM if there are some deviations from normality and mutual excess independency also [1, p.19].

Conditional-relative differences method [29, 30] is the improvement of the least square method. The sum of conditional-relative differences squares serves as the approximation measure here, see formula (3.7):

$$\sum_{i=0}^{n} \frac{(y_{rasch_i} - y_{zad_i})^2}{{y_{zad_i}}^2} \to \min \quad , \qquad (3.7)$$

where y_{rasch_i} is the desired value, y_{zad_i}– value specified by the source test data.

Application of conditional-relative differences method allows increasing the approximation accuracy in small values area y_i, but decreases it in large values area y. Thus, the described method has some advantages at solving some specific tasks but

it we cannot recommend to apply it widely due to the computational complexity and obscurity of statistical properties of suchlike values [28, p.696].

In spite of the described advantages of the maximum likelihood method, there is a number of estimating problems in which it is hard to apply it because of significant mathematical or computational difficulties in finding the $L(\theta)$ maximum. In such cases, the moment method is often used; it has no asymptotic optimality properties but it often leads to relatively simple computations.

The moment method. If the parameters $\bar{\theta} = (\theta_1, \theta_2, \ldots \theta_M)^T$ are unknown, the initial distribution moments are the functions, see formula (3.8):

$$m_k(\bar{\theta}) = \int_{-\infty}^{\infty} y^k w\left(\frac{y}{\bar{\theta}}\right) dx . \qquad (3.8)$$

Basing on observations $y_1, y_2 \ldots y_n$ we may find the sample initial moments of k-th order $\hat{m}_k = \frac{1}{n}\sum_{i=1}^{n} y_i^k$, which serve as independent estimates of distribution moments $m_k(\hat{\theta})$. The moment methods consists in setting M sample moments equal to the corresponding distribution moments and in finding the estimates of unknown parameters from the equation system: $m_k(\theta_1, \theta_2, \ldots \theta_M) = \hat{m}_k$, $k = 1,2, \ldots M$. Besides the initial moments, we may use *the central distribution moments and sample central moments* for parameter evaluation, see formula (3.9):

$$\hat{\mu}_k = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{m}_1)^k . \qquad (3.9)$$

For some distributions, e.g. for a normal or exponential one, the parameter estimates obtained with the moment method align with the corresponding maximum likelihood estimate (MLE). At the same time, there are many problems, in solution of which the moment method application leads to obtaining less accurate estimates than the application of maximum likelihood method. Estimating the θ parameter – the even distribution expressed in correlation - is its case study:

$$w\left(\frac{y}{\hat{\theta}}\right) = \frac{1}{\theta},\ 0 \le y_i \le \theta,\ i = 1,2,\ \ldots\ n\ . \qquad (3.10)$$

To find the estimate with the moment method, let us set the expectation (i.e. the first initial moment) $m_1(\theta) = \frac{\theta}{2}$ equal to the sample mean $m_k = \frac{1}{n}\sum_{i=1}^{n} y_i$. As a result we obtain an unbiased estimate, congruence (3.11):

$$\hat{\theta}(\bar{y}) = \frac{2}{n}\sum_{i=1}^{n} y_i \qquad (3.11)$$

having dispersion $M\left\{\left(\hat{\theta}(\bar{y}-\theta)\right)^2\right\} = \frac{\theta^2}{3n}$. It should be noted, that the obtained value is $(n+2)/3$ times as dispersion $D_\varepsilon = M\left\{\left(\hat{\theta}(\bar{y}-\theta)\right)^2\right\} = \left(\frac{n+1}{n}\right)^2 \int_0^\theta x^2\ w\ (x)\ dx - \theta^2 = \frac{\theta^2}{n\ (n+2)}$, defining the accuracy of the corrected estimate of maximum likelihood. *Except initial moments, it is possible to use central distribution moments and sample central moments for parameter estimation.*

The obtained estimates may be applied, for instance, at developing the noise type sorter in radar receiver, because the Weibull distribution describes a wide range of possible noises like receiver noises, clutter reflections, hydrometeors, etc.

Consequently, the motivation of searching for efficient estimates with the maximum likelihood method is evident. However, there are some cases in which it is difficult to find the solution of likelihood equations but possible to obtain proper estimates with the moment method application.

Wavelet is a class of family functions local in time and frequency, i.e. small quantities in which all the functions are derived from one by its timeline shifts and tensions, i.e. they go in turn; it allows analyzing various data frequency components. *Wavelet-analysis* is relatively new and powerful mathematical apparatus; it is widely used in classical sciences like analyzing and processing numerical series of physical, geophysical and other experiment or observation results, in identification or synthesis of signals of different nature and structure, applied for processing, compression, storing

and transferring of large data volumes including analogue pictures and videos. It is “a mathematical microscope” for accurate studying of inner structures of non-uniform signals and functions.

Wavelet transforms are usually divided into *discrete wavelet transforms* (DWT) and *continuous wavelet transforms* (CWT). Speaking about the application, DWT is usually applied for signal encoding, while CWT – for signal analysis. As a result, DWT is widely used in engineering and computer sciences and CWT in scientific researches.

In that way in the classical Fourier analysis we naturally use sines and cosines or the general complex exponential as basic functions, because the other orthogonal functions, e.g. orthogonal polynomials, are difficult in results interpretation.

In wavelet analysis there is a great choice of basic functions and it is being extended constantly, its application is simplified because the wavelet transforms are included in mathematical packages as standard applications. *All the wavelet transforms consider a function (taken as being a time function) in terms of oscillations localized in time and frequency*. Table 3.3 shows some of the main classical wavelet formulas.

Let us consider the main disadvantages of conventional Fourier analysis:

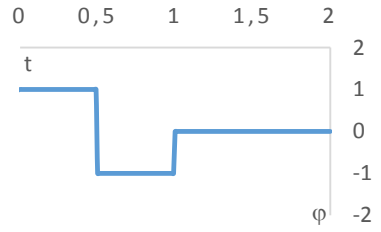
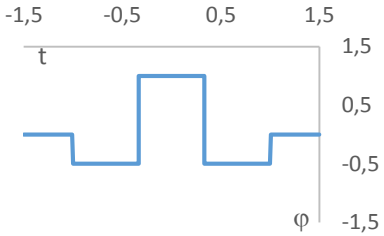
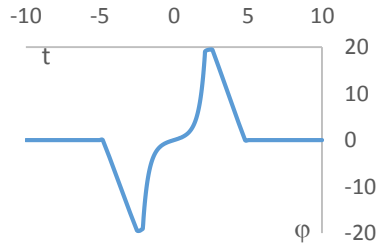
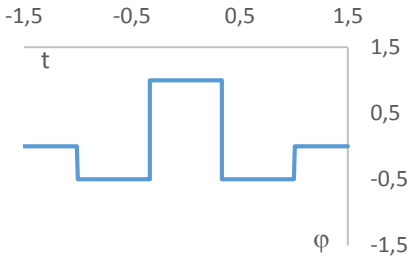
1. Insufficient informational contents at analyzing non-stationary signals and almost complete lack of opportunities to analyze its peculiarities or singular behavior and the appearance of destructive components absent in a source signal given that it had some disturbances and discontinuities;
2. Frequency basic expansion functions are basically incapable of reflecting the signal differentials having continued rate, e.g. rectangular pulses, because it requires infinite number of series terms, otherwise (at a finite number of Fourier series terms) strong oscillations appear in the neighborhood of disturbances and discontinuities in regenerated signals, i.e. *Gibbs phenomena*.
3. The Fourier transform provides the general information on the investigated signal frequencies and does not describe its local properties at fast time changes of its spectral distribution. The classical Fourier transform algorithm does not

provide an opportunity to analyze the signal frequency characteristics at random time moments (this information is not required for a stationary signal).

Besides there is a fundamental problem connected with the finiteness of the series investigated, while the Fourier transform requires a function to be specified and periodical at an infinite horizon.

Classical basic wavelets

Table 3.3

MATHEMATICAL WAVELET REPRESENTATION	GRAPHICAL WAVELET VIEW
Haar-wavelet: $\varphi(t) = \begin{cases} +1, & 0 \le t < 0{,}5, \\ -1, & 0{,}5 \le t < 1, \\ 0, & t < 0,\ t \ge 1. \end{cases}$	
Fhat-wavelet: $\varphi(t) = \begin{cases} +1, & \lvert t\rvert < \frac{1}{3}, \\ -0{,}5, & 1/3 < t \le 1, \\ 0, & \lvert t\rvert > 1. \end{cases}$	
Wawe-wavelet: $\varphi(t) = (t)\ exp\ (-\frac{t^2}{2})$	
Mhat-wavelet: $\varphi(t) = (1 - t^2)\ exp\ (-\frac{t^2}{2})$	

All the wavelet transforms may be considered as a variation of temporal-frequency representation and, consequently, refer to the subject of Fourier analysis.

Discrete wavelet transform may be considered as a variation of finite impulse response filter. CWT wavelets are governed by Heisenberg uncertainty principle, and consequently the discrete wavelet basis may be considered in the context of the other forms of uncertainty principle.

Nowadays wavelet transforms have a wide range of various applications, it often substitutes the conventional Fourier transform in many areas. Such paradigm shift occurs in many areas of physics, including molecular dynamics, ab initio computations, physical astronomy, localization of density matrix, seismic geophysics, optics, turbulence, quantum mechanics, image processing, blood pressure, pulse and electrocardiogram analysis, DNA analysis, protein researches, climate investigation, general signal processing, speech recognition, computer-generated images, multifractal analysis and others.

Methods of solving the problem of parameter signal identification by applying wavelets are at the initial stage of development now. It is a new area just starting its development - wavelet application at solving differential equations. G.Beylkin, Professor of state Colorado University, is the founder of representation theory of differential operators in *wavelet basis*.

We should note that it is unreasonable to use high degrees of interpolation polynomials at global approximation method. That is why patch-global method based on, for example, linear or quadratic (parabolic) interpolation is often used, but the derivatives of such interpolation polynomials obtained at the finite intervals and its joining points have discontinuities.

The considered approximating functions $S_m(x)$ (where m is a polynomial degree) should satisfy the requirements of continuity $S_m(x)$, $S'_m(x)$ and, if possible, $S''_m(x)$ or $S'''_m(x)$ at all the points $x \in [a;b]$, which gives rise to the necessity of developing spline-

functions having the above mentioned characteristics and being of interpolation or smoothing nature.

A spline-function, or a *spline,* is a class of $S_{m,i}(x)$ — algebraic polynomials of m order (components), defined at the finite intervals $[x_i, x_{i+1}]$, $[x_i, x_{i+1}]$, $i = \overline{0, n-1}$ and joined together in all the finite intervals in such a way that we can set up the multitier function, see equation (3.12):

$$S_m(x) = \bigcup_{i=0}^{n-1} S_{m,i}(x), \tag{3.12}$$

that is definite and continuous on the whole interval together with all of its derivatives $S_m^{(p)}(x)$ up to some of its order p=1,2... The difference in m and the highest derivative order continuous at the interval $[a,b]$ presents the *defect of spline q*. The conditions of matching the spline components $S_{m,i}(x)$ to a source function $y_i = f(x_i)$ at a corresponding finite interval $[x_i, x_{i+1}]$ are the conditions imposed on disparities of differential and integral types $\delta S_{m,i}^{(p)}(x_i)$, $\delta S_{m,i}(I_i^{i+1})$ and used for developing a formula of one spline component at a specified finite interval.

The number of matching conditions required for developing a formula of one spline component should correspond to the spline order, i.e. the number of conditions greater by unity than m. Having defined the formula $S_{m,i}(x)$, we express its right part through its known and unknown parameters: $f^{(p)}(x_i)$ for differential spline or through the set of $f^{(p)}(x_i)$ and $I_i^{i+1} = \int_{x_i}^{x_{i+1}} f(x)\,dx$ for integral-differential one. These values are called the spline parameters. Depending on the fact, if that or those spline parameters have been specified at the task setting or not, they are called definite or indefinite parameters. The computational basis for the last ones is *the continuity condition*, i.e. *the spline smoothness* $S_{m,i}(x)$ that are called *the connection conditions* here. For some juncture x_i, common to i-1 component referring to a finite interval $[x_{i-1}, x_i]$ and i component defined at a finite interval $[x_i, x_{i+1}]$, the condition looks as follows: $S_{m,i-1}^{(p)}(x_i)|_{x=x_i} = S_{m,i}^{(p)}(x_i)|_{x=x_i}$.

At solving the approximation tasks with spline functions, we transform the connection conditions to correlations connecting definite and indefinite parameters and called *parameter correlations*.

Parameter correlations written in the form of equations may be applied at finding the indefinite parameters out in relation to spline approximation or at expressing some parameters through the others. Besides, parameter correlations based on the principle of concordance of numerical quantity orders establish the correspondence of the orders for various parameters included in one correlation or another.

There is also a simpler but less strict method for developing empirical analytical dependences – the so-called mean-value method.

The core of *the mean-value method* is that the parameters of the required empirical formula are defined on the assumption that the sum of all the observed quantity deviations from its mean value is zero: $\sum_{i=1}^{n}(y_i - \bar{y}) = 0$, where $\bar{y}$ is the function mean value. "The method is the most efficient one in the simplest situation, when we need to develop the dependency of $y=b_1 x$ kind" [33, p.101].

In cases, where we have not managed to exclude gross outliers from the source data selection, it is better to use the special methods of robust regression.

One of such methods is the method of the least [38] that are squared if the LSM is applied. The principle of the method is minimizing the sum of moduli of differences between the desired values and the values specified by the source test data: $\sum_{i=1}^{n}\left|y_{rashc_i} - y_{zad_i}\right| \to min$, where y_{rasch_i} is the desired value, y_{zad_i} - the value specified by the source test data.

It should be noted that there are no methods of statistical analysis of LADM-regression equations. Whereas in the least square method the regression parameter distributions and their main statistical properties have already been studied, thus allowing performing the required statistical analysis, in the LADM the distributions remain unknown. So we know nothing about statistical properties of regression

obtained by the LADM-method. This is the substantial defect of the LADM method limiting its application in statistics [28, p.693].

One more approximation method is the uniform approximation method, in which the modulus of maximum deviation of the desired values from the specified test data serves as the approximation measure: $|y_{rasch} - y_{zad}|_{max} \to min$, where y_{rasch_i} is the desired value, y_{zad_i} - the value specified by the source test data. Such measure is also called the min-max approximation measure.

It is commonly supposed that the best uniform approximation provides a little better approximation than the least square one. However, theoretical studies show that such advantage is negligible; it is larger for the functions having discrete higher derivatives not too large in absolute value if the volume of source data is not large. However, the method has a substantial disadvantage – the absence of sufficient computational algorithms (except the direct search with non-linear programming methods) makes the given method hardly suitable for use.

There are other approximation methods – in particular, the extreme point method [28, p.694], *single-valued approximation method, method of inverted divided differences* [27].

At the next stage of forecast model development we need to choose its structure type – to sort out the class of functions. As in designing any math model of some real-world object, there are two approaches to model development – theoretical and empirical one [56].

Choosing the class of functions on the basis of theoretical approach enjoins taking into consideration the physical laws governing the investigated phenomenon provided by the results of experiments conducted.

It means that we develop the standard form of the analytical formula on account of some known scientific laws (laws of physics, chemistry, biology, etc.), and all the parameters included in the formula have some physical meaning. However, *developing a model even in the simplest cases may require applying several (not a single one) fundamental laws* [44, p.33].

At the same time we may represent the test data as an empirically fitted functional dependence with one or the other degree of correspondence to the observed phenomenon.

If we do not know the type of the dependence between x and y, "then the empirical formula has an unspecified form. It is preferable to use simple formulas having high accuracy. If there is no information concerning the intermediate data, it is usually supposed that the empirical function is of analytical type having a smooth curve and no discontinuity points. There is no uniform method for finding out the best type of formula corresponding to the experiment data" [12, p.80]. When trying the dependence general form one should make sure that the fitting function $f(x)$ has the same peculiar features as the investigated function $y(x)$. For example, if function $y(x)$ is an even function by its content, then function $f(x)$ should also be an even one, etc.; *it is very important to convey the function behavior at large and small values of x, probable changes of its sign and other substantial features correctly* [39, p.41].

The problem of finding a compromise between the model accuracy and simplicity usually arises at this stage: we need to choose such class of functions that satisfies "the definite correlation between the value characterizing the quality of function approximation to the set of source test data and the value characterizing "complexity" of approximating function [9, p.9].

While choosing the class of functions we may rely upon the 'from-simple-to-complex' modelling algorithm [60, p.6], *checking consequently various classes of functions from elementary to more complex ones.*

First we should check for the linearity of the required dependency, and then find out if there is any probable internal linearity. Sometimes an internally linear dependence hides behind the non-linear function structure in outward appearance. Such functions can be transformed to a linear form by change of the variables called linearization. The function linearization allows simplifying the computational procedure, because trying the parameters for a linear dependence is much easier from computational point of view than for a non-linear dependence. However, we should remember that the best choice of parameters for the transformed dependency may turn out to be not the best one for a source function. Besides, having computed the linear dependence parameters, we need to make a conversion calculation to get back to the source variables. Nevertheless, changing variables (the examples of linearizing transformations may be found in [25, p.415]) may help in finding the class of functions suitable for analytical representations of the source test data. Then we search for the suitable options in a set of non-linear functions. Among the classes of functions we may enumerate the functions described in Table 3.4.

Classes of mathematical functions

Table 3.4

MATHEMATICAL SPECIES OF FUNCTIONS	
1.	Class of functions: ***power functions*** *(elementary algebraic)* *Specific instance–**hyperbolic** and **parabolic***
2.	Class of functions: ***Exponential and logarithmic functions*** *(elementary analytical)* *Specific instance – **exponential***
3.	Class of functions: ***Periodic – line and arc-trigonometric functions*** *(elementary analytical)*
4.	Class of functions: ***rational*** *(elementary algebraic)* *Specific instance –**linear-fractional and polynomial functions***
5.	Class of functions: ***S-shaped curves and growth functions*** *(composite functions)*
6.	Class of functions: ***Bell curves and peak functions*** *(composite functions)*
7.	Class of functions: ***Delta functions*** *(composite functions)*

Let us note that, if ideally, the chosen type of function should completely reflect the physical meaning of the investigated phenomenon as much as all the parameters of the chosen functional dependence should be physically interpretable.

Polynomial dependency is convenient for describing the test data not only from the point of view of computational simplicity, but from opportunity to obtain an analytical equation having the specified accuracy that depends on the chosen polynomial order *n* directly. However, from the point of view of the required analytical dependence semantics, this class of functions has substantial disadvantages: indices in the obtained polynomial dependence usually have no real physical interpretation; and it has been proved that there is a whole class of functions that cannot be interpolated with a polynomial on the equally spaced grid of values. These are the functions having poles on the Mobius plane at a point of interpolation interval, e.g. a function having poles in $x = +i$ and $x = -i$ points. The larger *n* is, the more widely the interpolation polynomial will deviate from the function on drawing near the interval borders; at the unlimited growth of the number of points the interpolation error at the interval will go to infinity.

It should be understood that the growth of *interpolation error* at the growth of the number of interpolation points is not an algorithm problem and not a consequence of natural errors of real-numbers operations, but a fundamental property of interpolation polynomial because while passing through all the prescribed points, it rises sharply in the interval between them. As for the *errors of real-numbers operations*, they may be accumulated at the interpolation on the grid having equally spaced nodes and lead to the loss of interpolation quality. The reason is the following: even if the interpolated function refers to the "*well-behaved functions*" class, i.e. it has no poles at a point of interpolation interval, the errors of real-number operations usually add some contaminations to its graph. Such contaminations often assume the similitude to an "*ill-behaved function*" thus leading to a catastrophic error growth at the growth of point number.

The above-described problem has two solutions. If for some reason we cannot withdraw from the grid having equally spaced nodes, we may use *cubic splines* or *rational functions*. But if we are free in choosing the points, we may perform the interpolation on *Chebyshev grid* with *Chebyshev polynomials*, see formula (3.13):

$$Tk\,(x) = cos\,(k\,arccos\,x), \quad |x| \leqslant 1. \tag{3.13}$$

The Chebyshev polynomials are the least deviating from zero polynomials. In most cases on such a grid the interpolation error is reduced at the growth of number of points, particularly, it is true for any of the smooth functions. Computational errors are also less inclined to accumulation: the interpolation error is greater on the interpolation interval borders at the interpolation with equidistant nodes; but at the Chebyshev nodes interpolation the error is more uniformly distributed on the interpolation intervals and is smaller than at the interpolation with equidistant nodes.

Consequently, if it is difficult or impossible to fit an elementary class function for describing the source test data, we have to consider the classes of more composite functions, e.g. S-shaped curves and growth functions, bell curves and peak functions, Table 3.4. Typical graph of such-like functions is shown at Figure 3.1 and in the below-given example (article 3.5) of developing a forecast function for a wind velocity.

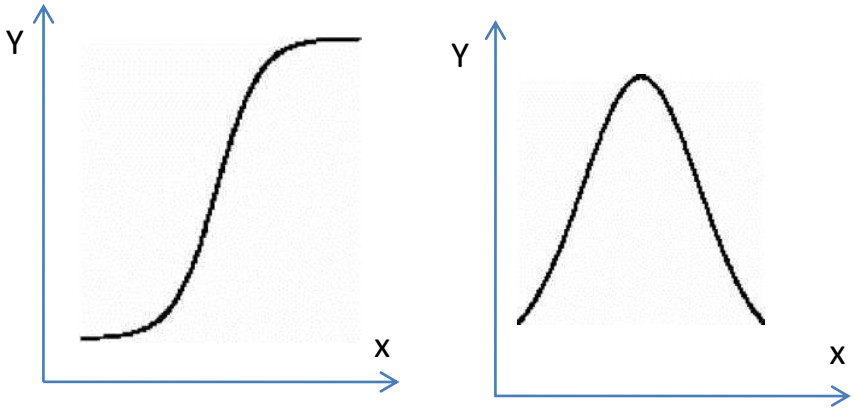


Figure 3.1 Functions graphs: a) s-shape curves and growth functions;
b) bell curves and peak functions

To the s-shape curves we refer, for instance: logistic curve $y = \frac{A_1 - A_2}{1+(x/x_0)^p} + A_2$, Boltzmann curve $y = \frac{A_1 - A_2}{1+\exp((x-x_0)/D)} + A_2$ and others.

To the bell curves and peak functions we refer, for instance, Gaussian function $y=y_0+\frac{A}{w\sqrt{\pi/2}}\cdot\exp(\frac{-2(x-x_c)^2}{w^2})$.

At the beginning of the XX century mathematics had no necessary strong descriptions for operating with a new class of value dependences discovered in physics; that is why the scientists introduced a new notion of *generalized functions*.

The need in suchlike generalization often appears in many physical and mathematical problems; the idea of generalized function indicates the fact that it is impossible in reality to measure the physical quantity value at some point and we can measure only its average values in a small neighborhood of the point. The derivative of δ-function is a generalized function also, and the integral is a Heaviside function.

So, the generalized function method is a convenient and adequate tool for describing the distribution of various physical quantities.

The list of the various functions can be found in *special atlases of* charts [57, 59].

The next research stage is calculating the parameter values of the functional dependence defined at the previous stage according to the chosen approximation method. If we do not manage to find the parameters, it means that the choice of the formula class or correspondingly the approximation method and measure may have been ineffectual.

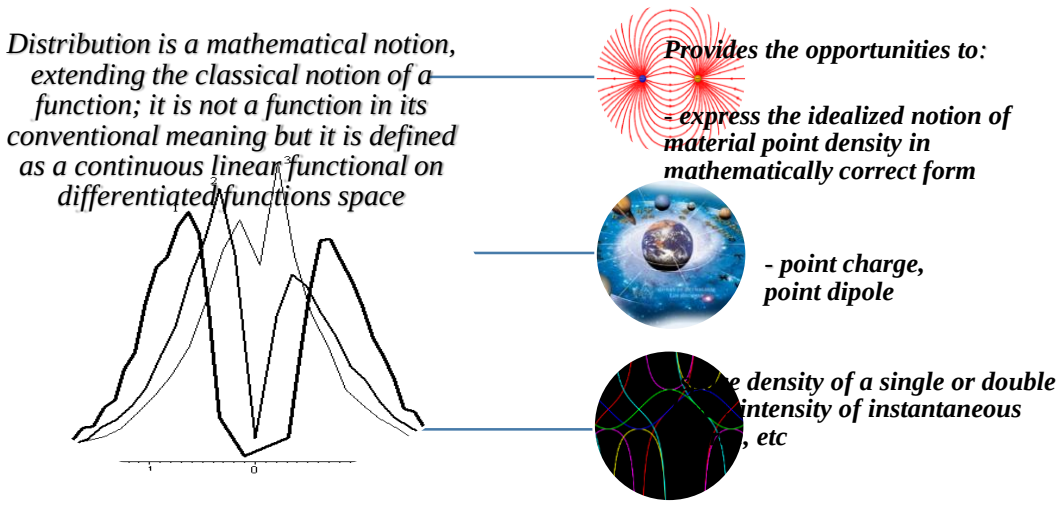


Figure 3.2 Generalized function description

3.3 Methods of analytical model investigation

Having developed an analytical dependence according to the experiment result, we need to choose the investigation method and establish the criterion of defining the correspondence of the obtained analytical model to a prior source data.

Classical function investigation methods involve the known methods of differential calculus, where an extreme point of the objective function $f(x)$ is found from the necessary condition of its existence. The condition is that the derivative at the extreme point is zero and the best solution of x^* can be found from the equation system (3.14):

$$\frac{\partial(x)}{\partial x_i} = 0\,, \quad \text{at } i = 1,2, \ldots, n. \tag{3.14}$$

To define if x^* is a maximum or minimum point, one should use the sufficient condition of extreme point existence, according to which: if at the extreme point the derivative changes sign from plus to minus, then $f(x^*)$ is an objective function maximum; if at the extreme point the derivative changes its sign from minus to plus, then $f(x^*)$ is an objective function minimum. If the given equations are non-linear, it is almost impossible to succeed in solving its system with analytical methods. In such cases, the ECM and corresponding numerical techniques or non-linear programming methods are used. In the last case, the problem of system solution is reduced to the problem of function minimization:

$$f\,(x) = \sum_{i=1}^{n} \left(\frac{\partial f\,(x)}{\partial x_i}\right)^2 . \tag{3.15}$$

The considered methods of investigating classical analytical functions may be applied at the solution of relatively simple optimization problems having no constraints. However, most of the engineering problems relate to optimization at some constraints of the controlled variables. Such constraints substantially reduce the size of the region on which the *optimum* search is conducted. On the face of it, it may appear that accessible region size reduction should simplify the optimum search. However, with the constraints, even the condition, according to which the optimum should be reached at a critical point characterized by a zero gradient, may be violated.

For example, unconstrained minimum of function $f(x) = (x\text{-}2)^2$ occurs at the stationary point $x = 2$. But if the minimization task is solved with regard to the constraint of $x \geq 4$, then we find the constrained minimum with the corresponding point $x = 4$. This point is not a stationary point of the $f(x)$ function, because f '(4).

Lagrange multipliers method allows finding the function maximum or minimum at the equality constraints. The basic idea of the method consists in moving from constrained extremum problem to the problem of finding the unconstrained extremum of some developed Lagrange function. Let us consider the general optimization problem containing several equality constraints: to minimize $f(x)$ at constraints $g_i = 0$, $j = 1, \ldots, k$. The problem may be solved basically as the unconstrained optimization problem obtained by elimination of independent variables with the help of the certain inequalities from the objective function k. The presence of the equality constraints allows in fact, at the constraint: $g(x) = x_1 + x_2 + x_3 - 1 = 0$. Having eliminated the variable x_3 with the equation $g(x) = 0$, we obtain the optimization problem having two variables and no constraints: $f(x_1, x_2) = x_1 \cdot x_2 + (1 - x_1 - x_2)$.

The elimination approach can be applied only in the cases when the equations expressing the restrictions may be solved for some definite set of the independent variables. If there are many equality constraints, the variable elimination procedure becomes rather a time-consuming one and sometimes the equations cannot be solved for variable.

In particular, if in the given example we set the constraints $g(x) = 0$ in the form of $g(x) = x_1^2 \cdot x_3 + x_2 \cdot x_3^2 + x_2^{-1} \cdot x_1$, then it is impossible to obtain an analytical expression of some of the variables through the other.

Consequently, it is reasonable to apply the Lagrange multipliers method at solving the optimization problems containing complicated equality constraints.

By applying this method, we may find the required conditions allowing identifying the optimum points in optimization problems with equality constraints.

Here the problem with equality constraints transforms to the equivalent unconstrained optimization problem. For example, the problem having several equality constraints: we minimize $f(x)$ at the constraints $(x) = 0$ at $j = 1,2 \ldots ,k$. According to the multipliers method, the problem translates to the following unconstrained optimization problem, the minimization is made as formula (3.16) shows:

$$L(x) = f(x) = \sum_{j=1}^{k} \vartheta_j \cdot g_j \,, \tag{3.16}$$

where $L(x, \vartheta)$ – Lagrange function, ϑ_j- Lagrange multiplier. There are no requirements to sign ϑ_j. We equate the partial derivatives $L(x, \vartheta)$ with respect to x to zero, obtain the following system of n-equations having n-unknown variables, see equation (3.17):

$$\frac{\partial L(x, \vartheta)}{\partial x_1} = 0, \ldots \frac{\partial L(x, \vartheta)}{\partial x_n} = 0. \tag{3.17}$$

Kuhn-Tucker conditions. The Lagrange multipliers method may be applied at developing the optimality criterion for problems with equality constraints. Kuhn and Thucker expanded this approach for the general problem of non-linear programming, one needs to minimize $f(x)$ at the constraints: $g(x) \geq 0$ $j=1,2,\ldots,I$ $h_k(x) = 0$, $k = 1,2, \ldots, K$, where $x = x_1 , x_2, \ldots , x_n$. Inequality constraint $g_j(x) \geq 0$ is called an active or connecting constraint at point $\bar{x}$ if $g_j(\bar{x}) = 0$, and inactive or non-connecting if $g_j(\bar{x}) > 0$, where $\bar{x}$ is an admissible point, i.e. satisfying all the constraints. If there is an opportunity to find the constraints that are inactive in the optimum point before solving the task, then these constraints may be excluded from the model thus reducing its size.

Kuhn and Thucker have developed the necessary and sufficient optimality conditions for non-linear programming problems on account of the assumption of differentiability of f , g_j , h_k functions. Consequently, the Kuhn-Thucker problem consists in finding the vectors $x_{(n\times1)}$, $U_{(1\times I)}$, $\vartheta_{(1\times k)}$ satisfying the following conditions:

$$\begin{cases} \nabla f(x) - \sum_{j=1}^{I} U_j \cdot \nabla g_j(x) - \sum_{j=1}^{I} \vartheta_k \cdot \nabla h_k(x) = 0 \,, \\ g_j(x) \geq 0 \qquad\qquad j = 1, 2, \ldots , I, \\ h_k(x) = 0 \qquad\qquad k = 1, 2, \ldots , K, \\ U_j \cdot g_j(x) = 0 \ , \\ U_j \geq 0 \quad . \end{cases} \tag{3.18}$$

Variational calculus. When solving some problems, we need to introduce the element modeling functions to describe the geometric properties of constantly changing cross sections of a mechanical system structural component. Let us consider two representative examples to define the attributes of the given problem class.

1. It is required to find the shortest curve between two points (x_0,z_0) and (x_1,z_1) in the plane *X* and *Z*. The length is put in correspondence to any curve connecting two specified points. The problem consists in choosing a curve *Z(x)* having the shortest length. For a curve *Z*(*x*) the length is defined by formula (3.19):

$$f(x) = \int (1 + (\frac{dz}{dx})^2)^{0,5} dx \, . \qquad (3.19)$$

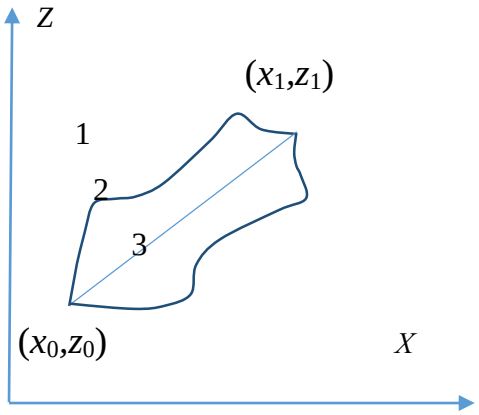


Figure 3.6 Illustrating example No.1

2. Two points (x_0,z_0) and (x_1,z_1) not lying on the vertical line are specified in the vertical plane. It is required to find such a curve connecting these points, in the line of which a particle having been at a standstill at (x_0,z_0) will slide frictionless to a point (x_1,z_1) at the shortest time.

Let *m* be a mass of a particle, *g* – free-falling acceleration; as the particle starts moving from a standstill (x_0,z_0) and the friction is missing, we may put the energy equation down as follows, see formula (3.20):

$$\frac{1}{2} m V^2 = m\, g\, (z - z_0) \, . \qquad (3.20)$$

where *V* is a velocity defined by formula (3.21):

$$V = ((\frac{dx}{dt})^2 + \left(\frac{dz}{dt}\right)^2)^{0,5} = (1 + (\frac{dz}{dx})^2)^{0,5} \frac{dx}{dt}, \quad (3.21)$$

where *t* is the time of particle moving.

3.4 Analyzing model parameters

Having found the parameter values in empiric formulas, we need to check the fidelity of the obtained analytical representation. For this purpose we calculate the deviation percentage for desired values obtained from the source test data on the basis of the defined analytical representation. The computational procedure for deviation percentage depends on the chosen fidelity criteria.

For example, the deviation percentage may be estimated as *the determination factor*.

If the value of deviation percentage is satisfactory, the analytical representation has been obtained and the algorithm has led to the problem solution.

If the deviation percentage outnumbers the allowable value or the parameter values have not been found at all, then probably some wrong dependence structure or approximation method has been chosen. In such a case, it is required to get back to the algorithm start and complete the next research iteration.

Having found that the computed fidelity criteria of the defined analytical model fit the initially specified limits, we may check for the model adequacy by conducting real-world experiments.

3.5 Example of forecast function development

Problem. It is required to develop a wind velocity forecast function for MPES in the nearest 10 minutes on the basis of the test data results shown at Table 3.5.

Test data results for the wind velocity

Table 3.5

TIME (h.min.sec)	VELOCITY (m/s)		TIME (h.min.sec)	VELOCITY (m/s)		TIME (h.min.sec)	VELOCITY (m/s)
0:00:00	6.09		0:02:55	1.66		0:05:35	4.29
0:00:05	6.33		0:03:00	3.09		0:05:40	4.15

0:00:10	5.88		0:03:05	2.51		0:05:45	2.92
0:00:15	4.97		0:03:10	2.04		0:05:50	1.61
0:00:20	5.69		0:03:15	4.3		0:05:55	4.09
0:00:25	5.37		0:03:20	8.13		0:06:00	4.69
0:00:30	5.62		0:03:25	8.86		0:06:05	3.29
0:00:35	5.12		0:03:30	2.39		0:06:10	2.51
0:00:40	4.43		0:03:35	0.34		0:06:15	4.04
0:00:45	3.04		0:03:40	1.05		0:06:20	1.99
0:00:50	4.16		0:03:45	0.23		0:06:25	4.43
0:00:55	3.88		0:03:50	0.96		0:06:30	3.36
0:01:00	2.82		0:03:55	3.00		0:06:35	4.80
0:01:05	2.16		0:04:00	3.94		0:06:40	3.76
0:01:10	3.17		0:04:05	5.92		0:06:45	5.37
0:01:15	2.09		0:04:10	1.93		0:06:50	4.1
0:01:20	1.59		0:04:15	1.91		0:06:55	0.70
0:01:25	2.71		0:04:05	5.92		0:07:00	1.54
0:01:30	4.97		0:04:10	1.93		0:07:05	1.28
0:01:35	3.22		0:04:15	1.91		0:07:10	0.47
0:01:40	1.5		0:04:20	2.41		0:07:15	0.56
0:01:45	0.80		0:04:25	6.11		0:07:20	0.53
0:01:50	0.87		0:04:30	5.49		0:07:25	1.09
0:01:55	2.18		0:04:35	7.25		0:07:30	0.75
0:02:00	1.86		0:04:40	10.28		0:07:35	0.46
0:02:05	0.81		0:04:45	9.31		0:07:40	0.53
0:02:10	4.15		0:04:50	8.07		0:07:45	1.35
0:02:15	5.29		0:04:55	9.75		0:07:50	1.01
0:02:20	6.96		0:05:00	7.82		0:07:55	1.58
0:02:25	5.56		0:05:05	5.6		0:08:00	3.13
0:02:30	5.65		0:05:10	2.33		0:08:05	6.31
0:02:35	4.47		0:05:15	1.38		0:08:10	0.71
0:02:40	1.38		0:05:20	0.51		0:08:15	0.73

0:02:45	1.25		0:05:25	2.56			
0:02:50	0.74		0:05:30	5.71			

Problem solution:

1) We introduce and define two array-variables: for the values of the time of occurrence and for the wind velocity values.

time: - ("0:00:05" "0:00:10" "0:00:15" "0:00:20" "0:00:25" "0:00:30" "0:00:35" "0:00:40" "0:00:45" "0:00:50" "0:00:55" "0:01:00" "0:01:05" "0:01:10" "0:01:15" "0:01:20" "0:01:25" "0:01:30" "0:01:35" "0:01:40" "0:01:45" "0:01:50" "0:01:55" "0:02:00" "0:02:05" "0:02:10" "0:02:15" "0:02:20" "0:02:25" "0:02:30" "0:02:35" "0:02:40" "0:02:45" "0:02:50" "0:02:55" "0:03:00""0:03:05" "0:03:10" "0:03:15" "0:03:20" "0:03:25" "0:03:30" "0:03:35" "0:03:40" "0:03:45" "0:03:50" "0:03:55" "0:04:00" "0:04:05" "0:04:10" "0:04:15" "0:04:20" "0:04:25" "0:04:30" "0:04:35" "0:04:40" "0:04:45" "0:04:50" "0:04:55" "0:05:00" "0:05:05" "0:05:10" "0:05:15" "0:05:20" "0:05:25" "0:05:30" "0:05:35" "0:05:40" "0:05:45" "0:05:50" "0:05:55" "0:06:00" "0:06:05" "0:06:10" "0:06:15" "0:06:20" "0:06:25" "0:06:30" "0:06:35" "0:06:40" "0:06:45" "0:06:50" "0:06:55" "0:07:00" "0:07:05" "0:07:10" "0:07:15" "0:07:20" "0:07:25" "0:07:30" "0:07:35" "0:07:40" "0:07:45" "0:07:50" "0:07:55" "0:08:00" "0:08:05" "0:08:10" "0:08:15").

values: - (6.09 6.33 5.88 4.97 5.69 5.37 5.62 5.12 4.43 3.04 4.16 3.88 2.82 2.16 3.17 2.09 1.59 2.71 4.97 3.22 1.55 0.80 0.87 2.18 1.86 0.81 4.15 5.29 6.96 5.56 5.65 4.47 1.38 1.25 0.74 1.66 3.09 2.51 2.04 4.30 8.13 8.86 2.39 0.34 1.05 0.23 0.96 3.00 3.94 5.92 1.93 1.91 2.41 6.11 5.49 7.25 10.28 9.31 8.07 9.75 7.82 5.60 2.33 1.38 0.51 2.56 5.71 4.29 4.15 2.92 1.61 4.09 4.69 3.29 2.51 4.04 1.99 4.43 3.36 4.80 3.76 5.37 4.10 0.70 1.54 1.28 0.47 0.56 0.53 1.09 0.75 0.46 0.53 1.35 1.01 1.58 3.13 6.31 0.71 0.73),

where **time** array *is an array of measurement occurrence time* and **values** array *is an array of wind velocity measurements occurred in the corresponding moment of time (table 3.5).*

$\mathbf{k} := \mathbf{0}..\mathbf{cols}(\mathbf{time}) - \mathbf{1}$

$$\mathbf{A_{0,k}} := \frac{\mathbf{hhmmss}(\mathbf{time_{0,k}})}{\mathbf{hhmmss}(\text{"00:00:01"})} - \frac{\mathbf{hhmmss}\,(\mathbf{tims_{0,0}})}{\mathbf{hhmmss}(\text{"00:00:01"})}$$

2) Let us range the values of time of wind velocity measurements occurrences: **0:00:00 = 0, 0:00:05 = 5, 0:00:10 = 10**, etc.

$$\mathrm{L}(\mathbf{x}, \mathbf{valuesx}, \mathbf{valuesy}) :$$

$$= \left\| \begin{array}{c} \mathbf{result} \leftarrow \mathbf{0} \\ \mathbf{for\ i} \in \mathbf{0}..\mathbf{cols}(\mathbf{valuesy}) - \mathbf{1} \\ \mathbf{basic} \leftarrow \mathbf{1} \\ \mathbf{for\ j} \in \mathbf{0}..\mathbf{cols}(\mathbf{valuesx}) - \mathbf{1} \\ \mathbf{basic} \leftarrow \mathbf{basic} \dfrac{\mathbf{x} - \mathbf{valuesx_{0,j}}}{\mathbf{valuesx_{0,i}} - \mathbf{valuesx_{0,j}}} \ \mathbf{if\ i} \neq \mathbf{j} \\ \mathbf{result} \leftarrow \mathbf{resuit} + \mathbf{basic} * \mathbf{valuesy_{0,j}} \\ \mathbf{retum\ result} \end{array} \right.$$

3) Let us define a function computing the interpolation Lagrange polynomial on account of data arrays: **time** *and* **values**, *farther denoted as* **valuesx** *and* **valuesy** *respectively.*

4) Let us develop a graph of Lagrange polynomial where the Runge phenomenon occurs, i.e. the large outliers happen at high polynomial orders, see Figures 3.7, 3.8.

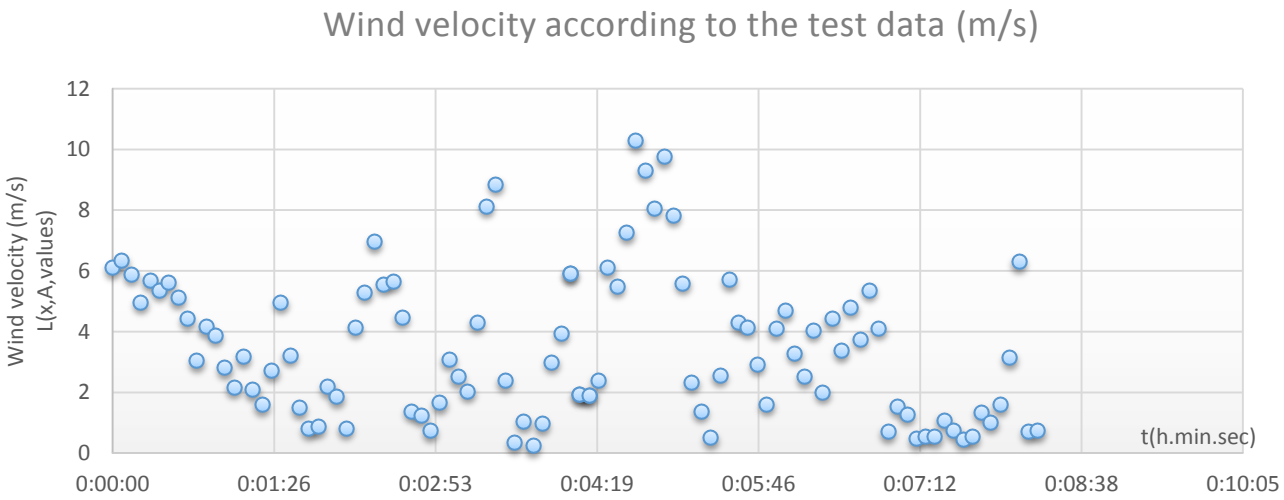


Figure 2.7 Graphical representation of Lagrange polynomial – Runge phenomenon observation

5) In connection of item 4, we need to choose the other method (LSM) and try the most suitable indices of power polynomial: by searching of **p** order values. *The*

experiments show that the 4th order polynomial allows achieving the minimum deviation.

$$\mathbf{LSM(c)} := \sum_{j=0}^{\mathbf{cols(values)}-1} \left[\mathbf{values_{0,j}} - \sum_{i=0}^{\mathbf{cols(c)}-1} \left[\mathbf{c_{0,i}} * \left(\mathbf{A_{0,j}} \right)^{i} \right] \right]^2$$

$$p := 0..2$$

$$\mathbf{CTest_{0,p}} := 0 \qquad \mathbf{F(CTest, x)} := \sum_{i=0}^{\mathbf{cols(CTest)}-1} \left(\mathbf{CTest_{0,i}} * \mathbf{x}^{i} \right)$$

6) Searching for indices by the LSM: searching for such a value of **CTest** *variable, at which the LSM function possesses the least value*

$$\mathbf{f(values, x)} := \left| \begin{array}{c} \mathbf{result} \leftarrow 0 \\ \mathbf{for\ i} \in 0..98 \\ \mathbf{result} \leftarrow \mathbf{values_{0,i}\ if\ x} = \mathbf{A_{0,i}} \end{array} \right.$$

$$\mathbf{F(Minimize(LSM, CTest), x)}$$

$$\mathbf{f(values, x)}$$

$$\mathbf{ooo}$$

7) Graphical representation of the test data **f(values, x)** *and approximating function* **F(Minimize(LSM, CTest),x)**, *see Figure* 3.8.

8) To exclude the outliers and obtain more correct results we apply the exponential smoothing method to the data: $i := 0..98 \quad j := 0..1 \quad a_i := \frac{1}{7}$

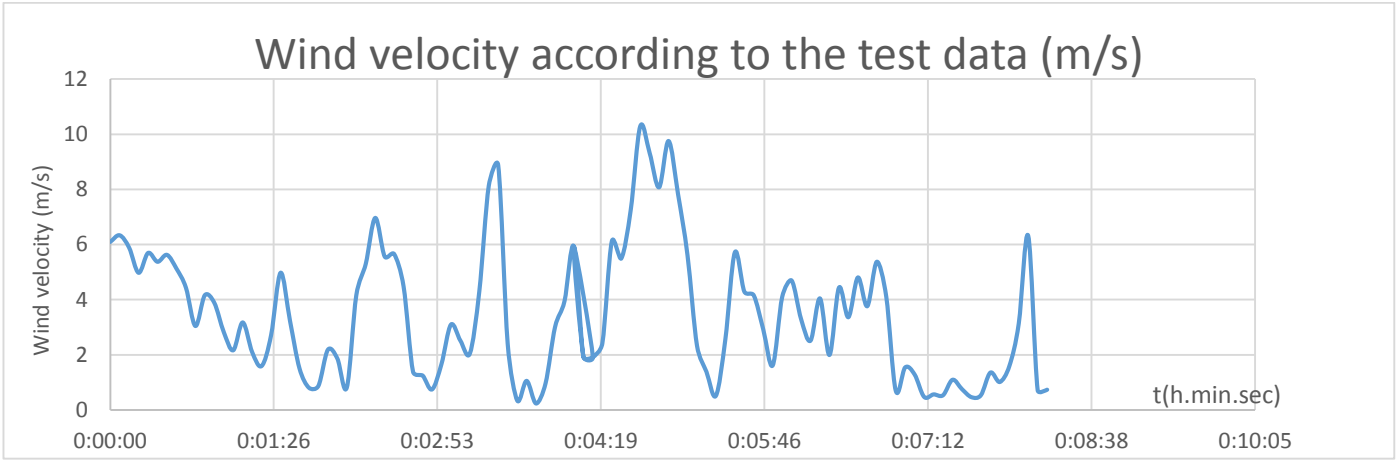


Figure 3.8 Graph of test data distribution

9) Searching for indices by the LSM: searching for such a value of **CTest** *variable, at which the LSM function possesses the least value.*

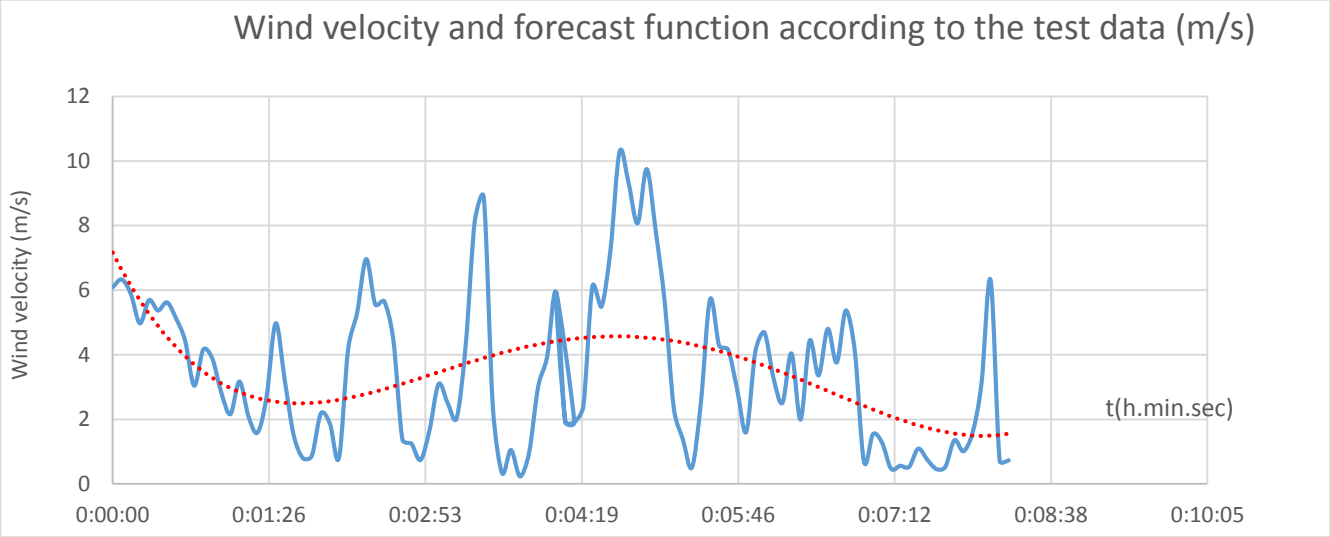


Figure 3.9 Forecast function graph – 4th order polynomial

10) *Graphical representation of the smoothed test data* **F(Smooth, x)** *and function* **F(Minimize(LSM, CTest), x)** *approximating it.*

The conclusion drawn according to the experiment results:

In the present paper we obtained the 4th order polynomial that interpolates and extrapolates the source data. Unfortunately, the discovered polynomial does not take into account the nature of the data, for which cause the extrapolation by the given polynomial may be conducted with significant errors.

CHAPTER 4. GENERALIZATION PROCEDURES FOR A FORECAST MODEL

4.1 Concept of developing an analytical forecast model of a technical process in the form of a component composition

Let us suppose that a technical process modelling combined with various conditions for which the experiments were made forms some U system that can be described through characteristic features of its elements. Let the given U system may be represented as sets shown at Figure 4.1, that is: a set of input parameters $\{V_i\}_1^{N_{Input}}$, medium external disturbances $\{W_i\}_1^{N_{Ext\_dist}}$, internal parameters $\{G_i\}_1^{N_{System}}$ and output parameters $\{Z_i\}_1^{N_{Exit}}$.

One of the output parameters, e.g. z_1, is our required parameter, the others (-z_2, ..., z_n-) should be considered at developing the system analytical model only if there is a feedback from them. In general, the forecast model of technical process looks as follows (Figure 4.1):

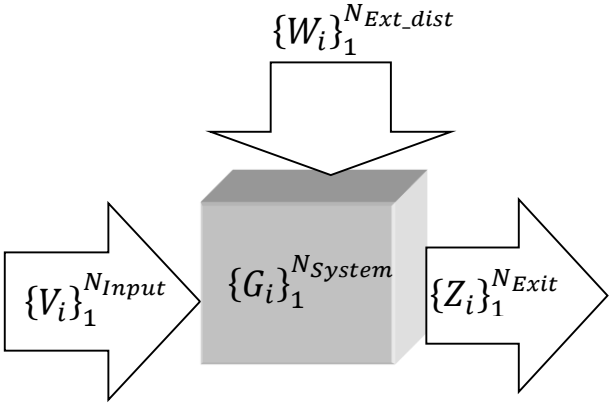


Figure 4.1 General scheme of U system functioning in actual

$$Z\left(x_1, x_2, ..., x_{N_{Выход}}\right) = V\left((x_1, x_2, ..., x_{N_{Вход}}) \circ W(x_1, x_2, ..., x_{N_{Возм}})\right) \circ G\left(x_1, x_2, ..., x_{N_{Систем}}\right) \quad (4.1)$$

To solve the specified problem it is reasonable to use the method of the analytical transformation of a function into a Fourier series that subsequently allows making

transparent the application of the following mathematical operations: differentiation, integrating, argument shift and convolution.

There are three approaches to represent mathematically the signals processed by an engineering system. *Approach one* – through the real signals expressed as a time function that are visible in oscillograph. Let us suppose that there is an arbitrary time function *S(t)* at a finite interval *(t_1, t_2)* and there is an orthogonal function system $\varphi_1(t), \varphi_2(t), \dots, \varphi_m(t)$, i.e.

$$\int_{t_1}^{t_2} \varphi_i(t)\varphi_j(t)dt = \begin{cases} 0, & i \neq j, \\ C_j, & i = j. \end{cases} \tag{4.2}$$

It has been affirmed that $S(t) = \sum_j S_{ij}\,\varphi_j(t)$, where

$$S_j = \frac{1}{k_j}\int_{t_1}^{t_2} S(t)\varphi_j(t)dt, \quad k_j = \int_{t_1}^{t_2} \varphi_j^2(t)dt. \tag{4.3}$$

It means that a function specified at a finite time interval (including a periodic function) can be represented in a series in a system of orthogonal functions. Trigonometric functions *sin*(*kt)* and *cos*(*kt)* are used as orthogonal functions system most frequently. Then, as a result of mathematical transformations, we obtain the following Fourier trigonometric series:

$$s(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k cos\ kt + b_k \sin kt), \tag{4.4}$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} s(t)\cos kt\ dt, \quad b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} s(t)\sin kt\ dt, \tag{4.5}$$

where $k = 0,1,2$.

If apply Euler formulas and proceed to complex variables, we obtain an exponential form of a Fourier series:

$$s(t) = \sum_{k=\infty}^{\infty} v_k \mathrm{e}^{jkt}, \quad v_k =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} s(t)\,\mathrm{e}^{-jkt}\ dt\ , \quad \mathrm{e}^{jkt} = \cos kt + i\sin kt\ , \tag{4.6}$$

where a_k , b_k or v_k the indices set is *a signal amplitude spectrum.*

As most part of real-world signals processed by an engineering system requires its specification at an unlimited time interval and they are not periodical ones, then *the second approach* to its mathematical representation is to distribute it in the form of a signal spectrum. *Forward and inverse Fourier transforms or the spectrum density* are applied to its representation*:*

$$V(\omega) = \int_{-\infty}^{\infty} v(t)\, \mathrm{e}^{-j\omega t}\ dt \tag{4.7}$$

- $v(t)$ function spectrum or the forward Fourier transform.

$$v(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} V(\omega)\, \mathrm{e}^{-j\omega t}\ d\omega \tag{4.8}$$

- Inverse Fourier transform.

Consequently, a signal may be specified either as a time function (time distribution) or as its function spectrum (frequency distribution).

The third approach for the mathematical signal specification is described in details with *the sampling theorem.* The approach allows considering the transfer of any message (continuous or discrete one) from a single position as a number transfer (or data transfer), i.e. the theorem specifies the opportunity of a complete recovery of a determinated band-limited signal by its sampling and designates the extreme values of between-sampling time at which its recovery is still possible.

The sampling theorem: if a continuous function x(t) satisfies the Dirichlet conditions (it is confined, sectionally continuous and has a finite number of extreme points) and its spectrum is limited with some frequency f_c, then it is defined completely by the sequence of its values at the points spaced from each other $\frac{1}{2} f_c$ apart.

Hence, the continuous signal may be specified digitally – as a sequence of transient values. Consequently, it is possible to have a single integrated signal for the

whole class of the considered engineering systems in the output by addressing data through the shared memory.

If we consider the sampling theorem as an approximate one for functions having unlimited spectrum, then the contradiction concerning the incapability of a model of limited spectrum signals to reflect the main signal property – *the ability to transfer the information* – is avoidable. The reason is the theoretical opportunity to forecast the limited spectrum function behavior at the entire temporal axis, if it is known with precision at the indefinitely small time interval.

At restoring the analogue signal, some errors appear caused by the constrained limitation of a signal spectrum and the finiteness of the number of samplings applied. These errors are estimated by special calculations that are in details described in Chapter 5.

Let us represent an analytical forecast model as a composition of three components shown at Figure 4.2 and described in Table 4.1.

$$z_1 = f_1\,(g_1, g_2, \dots, g_k) \circ f_2\,(v_1, v_2, \dots v_l) \circ f_3\,(w_1, w_2, \dots, w_m) = f_4\,(z_2, \dots, z_n), \qquad (4.9)$$

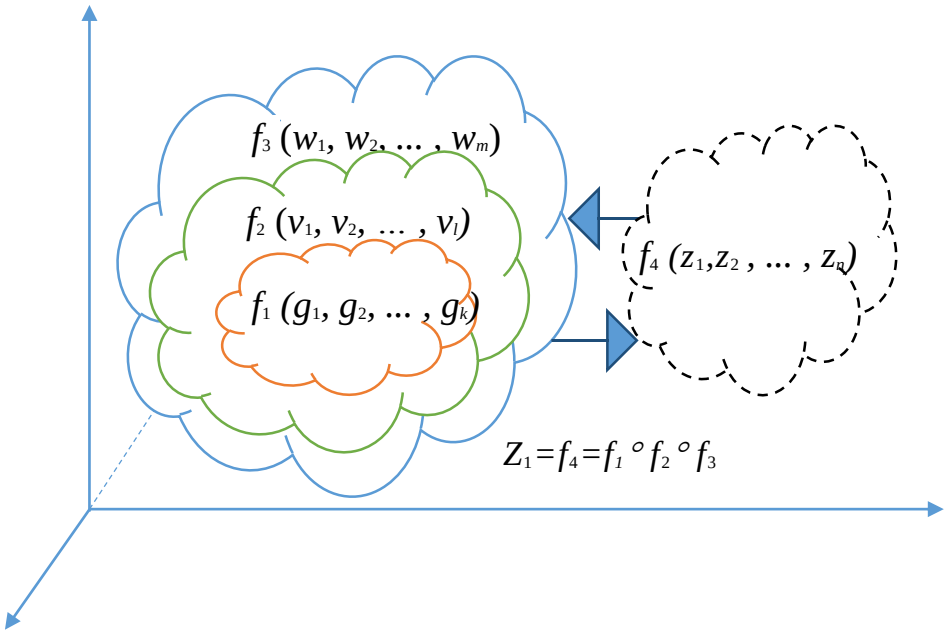


Figure 4.2 Standard form of the MPES analytical forecast function

Forecast model components

Table 4.1

FORECAST MODEL COMPONENTS	ACTING $\overline{F}$, DESCRIBED BY THIS COMPONENT	MODELLING STAGE CHARACTERISTIC
First component- $f_1(g_1, g_2, \dots, g_k)$ general (universal) part of the required model	The influence of the *U* system internal parameters, i.e. the description of $\overline{F}_{вн.}$ process;	Considering the process semantics and extracting the forecast function base;
Second and third components- $f_2(v_1, v_2, \dots, v_l)$ $f_3(w_1, w_2, \dots, w_m)$	The influence of the *U* system external parameters, i.e. the description of $\overline{F}_{внеш.}$ process and medium disturbances;	Declaring the fundamental limitations of the required function;
Fourth component- $f_4(z_1, z_2, \dots, z_n)$	Taking account of the feedback value, if there is any;	Opportunities for parametrical extension while choosing parameters to be taken into account and for control of the forecast and estimate accuracy with various degree of detail.

Interconnections between the model components can be expressed through the *correctness passage* of the required *function:* the smaller the value distinguishing one function component from the other neighboring one is, the higher is the probability that the function base have been extracted correctly. Mathematical correctness passage may be written as follows:

$$\begin{cases} f_1 \neq 0\ (otherwise \nexists\ process), \\ f_1 \neq f_2 \neq f_3 \left(acting\ \overline{F} - of\ different\ physical\ nature\right), \\ 0 < |f_1 - f_2| < 1, 0 < |f_2 - f_3| < 1, \end{cases} \quad (4.10)$$

where the upper border equal to 1 is in accordance with 100% correspondence (match) of the neighboring components of the required function.

The composition of the offered function parameters may be an additive, multiplicative or any other arbitrary dependence and be determined by the availability of continued and variable factors, i.e. the given function is a multifactorial function in itself:

$$f(x) = f_1(x) \circ f_2(x), \tag{4.11}$$

where $f_1(x)$ component characterizes the influence of the continued factors, and $f_2(x)$ accounts for the influence of the variable factors. For example, the continued factor for the atmosphere radiation transmission phenomenon is the radiation absorption with some gases (in particular, with carbon dioxide gas in case of the infrared radiation). Some gases in the atmosphere composition may vary depending on the current geographical and climatic location, and that is why we introduce additional component $f_2(x)$.

Besides, *additional components introduced to a model may not only account for the current environmental conditions but vary depending on the modeling aims*. It means that some regulating component except the basic and specifying ones may be introduced, for example, to estimate the worse or the best or some abnormal scenarios of the physical phenomenon investigated.

Representing the analytical model as a component composition allows obtaining the investigated phenomenon models having various degree of detail. Depending on the definite aims and conditions of the required model application, we may take into account or, on the contrary, exclude from consideration some components of the model structure, thus accounting for or setting certain influences aside.

Consequently, by correcting the result through the experiment, it is possible to transform it into the forecast model, and change of the model parameters at keeping its semantic meaning brings the model *weighting factors* into existence.

4.2 Base component of the MPES analytical forecast model

To extract the base or, in other words, the general part of an engineering process forecast function we investigate the oscillations of different nature, i.e. we take the universal part of oscillations as $f_1(g_1, g_2,..., g_k)$, see Figure 4.3.

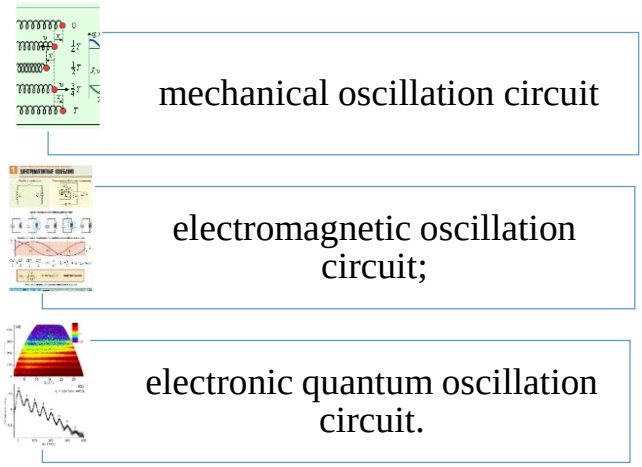


Figure 4.3 Oscillation of different nature

Let us consider a possibility that all the oscillations may be described with one of the main equations of mathematical physics, *a wave equation*, i.e. a partial differential equation that describes the disturbance propagation in some medium. If the disturbances are small and the exterior medium is a homogeneous-isotropic medium, then we can write it as follows:

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = \frac{1}{a^2}\frac{d^2u}{dt^2}, \tag{4.12}$$

where *x, y, z are space variables, t is disturbance travel time, u= u(x, y, z) is the required function that characterizes the disturbance in (x,y,z) point at t moment, a is disturbance velocity.*

If *u* depends on two or one space variables only, we are concerned with the simplified wave equation – two-dimensional or one-dimensional one.

Next we consider a solution of the wave equation not as corresponding to plane waves with its superimposition description (e.g. for electromagnetic oscillation) but in

the other mathematical form – in waves corresponding to the spherical surface that fall apart in three dimensions, i.e. through a spherical wave model.

The wave equation admits a "spherical wave spreading" solution; the variable characterizing the field takes the following form:

$$u = \frac{f(t-r/a)}{r}, \tag{4.13}$$

where f - is an arbitrary function,

$$r=\sqrt{x^2 + y^2 + z^2}. \tag{4.14}$$

Suchlike function represents *the spherical wave of general kind* that spreads from the origin of coordinates having velocity *a*. If we do not account for *r* in formula (4.13) consequent, then the wave amplitude as a distance function from the origin of coordinates has its definite form in every given moment *t* and it is spread with velocity *a*. However, *r* in the consequent shows that while the wave is spreading, its amplitude decays in proportion to 1/*r*. In other words, in contrast to a plane wave which amplitude does not change while running, the amplitude of a spherical wave is continuously decaying, see Figure 4.4. The fact follows from simple physical arguments, i.e. from the semantics of the physical process itself.

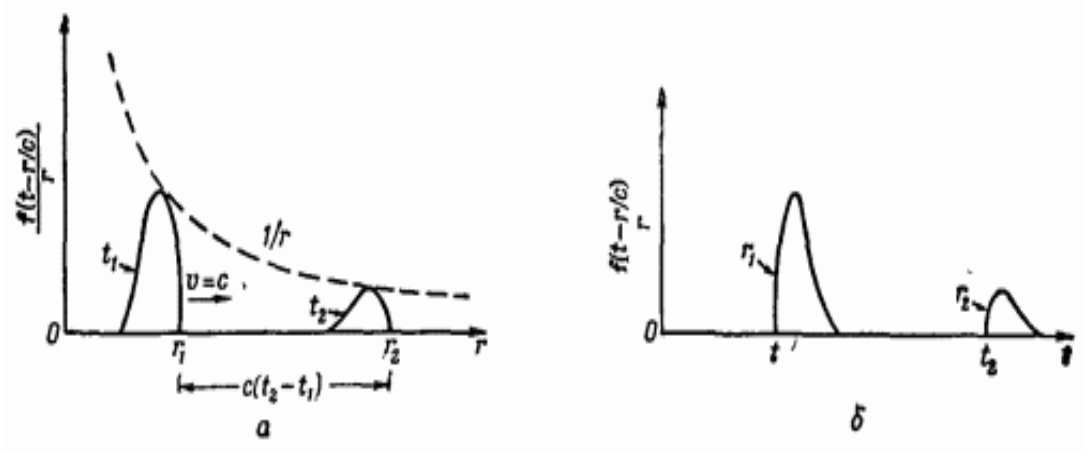


Figure 4.4 Spherical wave u= (f(t-r/a))/r

Figure 5 shows under the letter a) the r-dependence of u at $t=t_1$ and the same wave at a later moment t_2; under the letter b) the t-dependence of u at $r=r_1$ and the same wave at distance r_2.

The wave power density depends on the square of the wave amplitude: while the wave is running, its energy is spreading over to larger area in proportion to the wave radius square. If the full energy is kept, the energy density decays as $1/r^2$, and the amplitude as $1/r$. It follows from the above given argument that formula (4.13) is quite a "reasonable" and acceptable formula for applying it to a spherical wave.

The other possible solution for a one-dimensional wave equation is a spherical wave running inside from larger r to the origin of coordinates:

$$u = \frac{g(t-\frac{r}{a})}{r}. \tag{4.15}$$

Consequently, on account of the semantics of the process of spreading the spherical waves itself, we may assume that the waves generated by some source are always running away from it only. As the waves are caused by the charge movement only, we may logically assume that they are running away from the charges. So, thinking by contradiction, we assume the following: before the charges were set to move, the spherical wave had already escaped from infinity and come to the charges exactly when they started to move. The described solution is possible but it is hard to match such an argument with the logic of the given phenomenon because the experience shows that when the charges accelerate, the waves are moving away from the charges but not towards them. Although the electromagnetic field equations provide equal opportunities to both waves, one should invoke an additive factor based on the experience that only the spreading waves have physical sense.

The spreading waves system providing a virtual image is indistinguishable from the waves emitted by the object itself; its holograph fully reconstructs the object three-dimensional structure and shows the visible spacing of the items with parallax effect that consists of changing the visible relative item spacing at changing the observation point, *i.e. observing the spreading wave is equal to observing the object itself.*

Simplified wave equation of physical phenomena

Table 4.2

OSCILLATION TYPE	MATHEMATICAL EQUATIONS OF OSCILLATIONS
1. Mechanical	$\frac{d^2x(t)}{dt^2} + \omega_0{}^2 x(t) = 0$
2. Acoustic wave	$\frac{d^2x}{dx^2} = \frac{1}{c_s^2}\frac{d^2x}{dt^2}$
3. Vibration of a string	$\frac{d^2u}{dx^2} = \frac{1}{a^2}\frac{d^2u}{dt^2}$, where $u = f(t-r/a)/r$
4. Electromagnetic	$\frac{d^2q(t)}{dt^2} + \omega_0{}^2 q(t) = 0$

Let us consider an option of determining the base of a forecast function describing the real-time engineering system behavior by making comparison between mechanical and electromagnetic oscillations, see Figure 4.5 and Tables 4.3, 4.4.

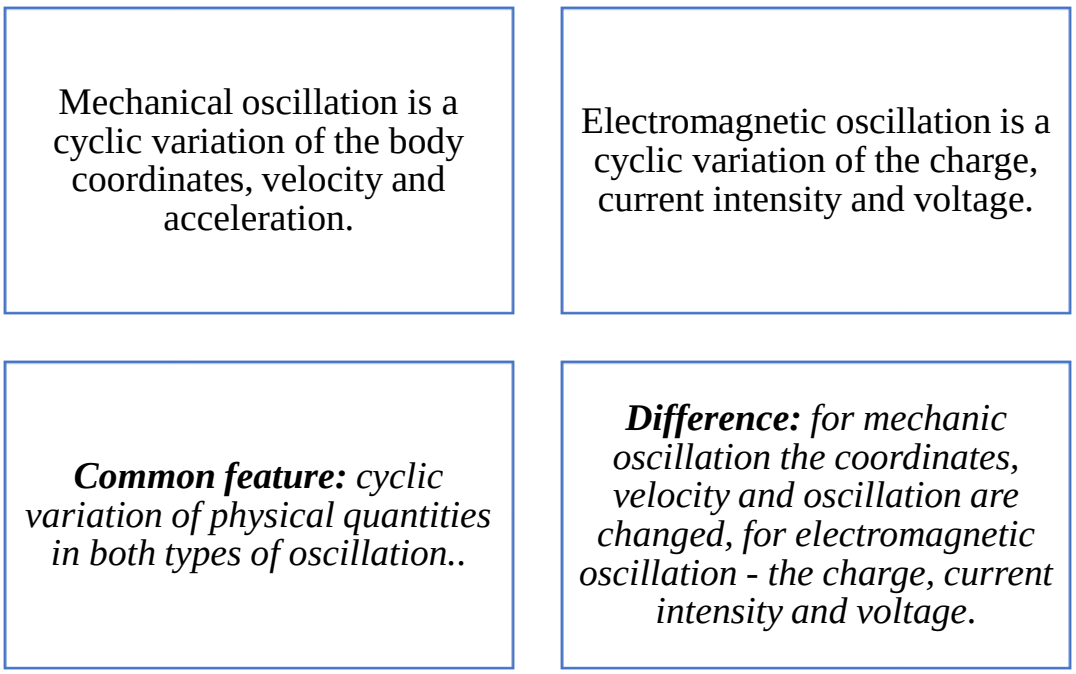


Figure 4.5 Comparison between mechanical and electromagnetic oscillation

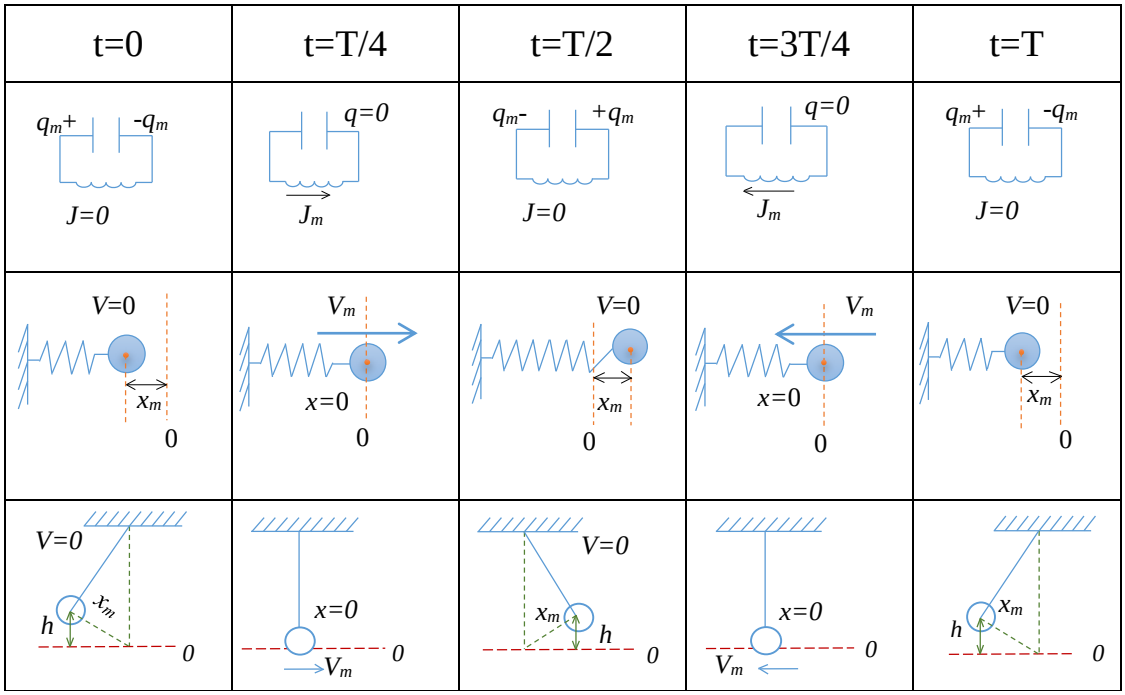


Figure 4.6 Comparison between the oscillating systems: mechanical and electromagnetic ones

Comparison between two oscillating systems: mechanical and electromagnetic ones

Table 4.3

TIME	OSCILLATING CIRCUIT	SPRING PENDULUM
$t = 0$	The capacitor has maximum charge q_m, $I = 0, W_{эл} = \max = \frac{q_m^2}{2c}, W_м = 0$	The body has maximum displacement from equilibrium x_m, $\mathrm{v} = 0, \mathrm{E}_\Pi = \max = \frac{kx_m^2}{2}, \mathrm{E}_\mathrm{K} = 0$
$0 < t < \frac{T}{4}$	At circuit closing, the capacitor starts running down through the coil, the current and the connected magnetic field appear; the current force increases gradually due to self-inductance $I \uparrow, q \downarrow \Rightarrow W_{эл} \to W_м$	The body sets to move, its velocity increases gradually due to the body inertia $v \uparrow, x \downarrow \Rightarrow E_n \to E_к$
$t = \frac{T}{4}$	The capacitor has run down, the current force is a maximum – I_m, $q = 0, W_{эл} = 0, W_м = \max = \frac{LI_m^2}{2}$	On passing through the equilibrium, the body velocity is a maximum – $\mathrm{v_m}$, $\mathrm{x} = 0, \mathrm{E}_\Pi = 0, \mathrm{E}_\mathrm{к} = \max = \frac{\mathrm{mv}_\mathrm{m}^2}{2}$

$\frac{T}{4} < t < \frac{T}{2}$	Due to self-inductance the current force is reduced gradually, the inductive current appears in the coil and the capacitor starts recharging $q\uparrow, I\downarrow \Rightarrow W_м \to W_{эл}$	Having come to the equilibrium, the body continues inertial motion gradually reducing the velocity $x\uparrow, v\downarrow \Rightarrow E_к \to E_n$
$t = \frac{T}{2}$	The capacitor has been recharged, the armature charge signs have changed $q = \max, I = 0,$ $W_{эл} = \max = \frac{q_m^2}{2c}, W_м = 0$	The spring is at maximum extension, the body has moved to the other side $\mathrm{x} = \max, \mathrm{v} = 0,$ $\mathrm{E_п} = \max = \frac{\mathrm{kx_m^2}}{2}, \mathrm{E_к} = 0$
$\frac{T}{2} < t < \frac{3T}{4}$	The capacitor restarts charging, the current flows in other direction, the current force increases gradually $I\uparrow, q\downarrow \Rightarrow W_{эл} \to W_м$	The body starts moving in the opposite direction, the velocity increases gradually $v\uparrow, x\downarrow \Rightarrow E_n \to E_к$
$t = \frac{3T}{4}$	The capacitor has run down, the current force in the chain is a maximum - I_m $q = 0, W_{эл} = 0, W_м = \max = \frac{LI_m^2}{2}$	The body passes through the equilibrium, its velocity is a maximum - v_m $\mathrm{x} = 0, \mathrm{E_п} = 0, \mathrm{E_к} = \max = \frac{\mathrm{mv_m^2}}{2}$
$\frac{3T}{4} < t < T$	Due to self-inductance the current continues flowing in the same direction, the capacitor starts charging $q\uparrow, I\downarrow \Rightarrow W_м \to W_{эл}$	The body continues inertial motion in the same direction to the extreme position $x\uparrow, v\downarrow \Rightarrow E_к \to E_n$
$t = T$	The capacitor is charged again, there is no current in the chain, the circuit state is similar to the initial $q = \max, I = 0,$ one $W_{эл} = \max = \frac{q_m^2}{2c}, W_м = 0$	The body displacement is a maximum, its velocity is 0 and its state is similar to the initial one $\mathrm{x} = \max, \mathrm{v} = 0,$ $\mathrm{E_п} = \max = \frac{\mathrm{kx_m^2}}{2}, \mathrm{E_к} = 0$

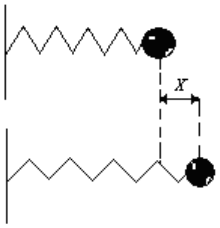
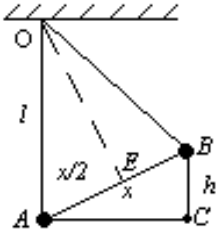
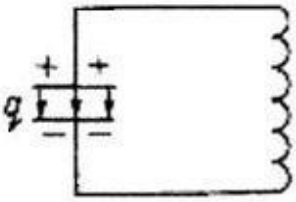
Corresponding values: mechanical and electromagnetic oscillations

Table 4.3

MECHANICAL OSCILLATIONS	CORRESPONDING VALUES	ELECTROMAGNETIC OSCILLATIONS
Displacement	$x \leftrightarrow q$	Charge
Velocity	$V \leftrightarrow I$	Current intensity
Mass	$m \leftrightarrow l$	Inductance
Rigidity	$k \leftrightarrow \frac{1}{C}$	Reciprocal of capacitance
Friction ratio	$\mu \leftrightarrow R$	Resistance
Kinetic energy	$E_к \leftrightarrow W_м$	Magnetic-field energy
Potential energy Mass Rigidity	$E_п \leftrightarrow W_э$	Electric field energy Inductance Reciprocal of capacitance

Oscillating process equation

Table 4.4

SPRING PENDULUM	SIMPLE PENDULUM	OSCILLATION CIRCUIT
	 Let us express h in terms of x through the similarity of ΔAOE and ΔABC $\frac{h}{x/2}=\frac{x}{l}$ $h=\frac{x^2}{2l}$	
$W_n = const$	$W_n = const$	$W_n = const$
$W_n' = (const)' = 0$	$W_n' = (const)' = 0$	$W_n' = (const)' = 0$
$(\frac{mv^2}{2}+\frac{kx^2}{2})' = 0$	$(\frac{mv^2}{2}+mg\frac{x^2}{2\ l})' = 0$	$(\frac{Li^2}{2}+\frac{q^2}{2C})' = 0$
$\frac{m}{2}2vv'+\frac{k}{2}2xx' = 0$	$\frac{m}{2}2vv'+\frac{mg}{2l}2xx' = 0$	$\frac{i}{2}2ii'+\frac{1}{2C}2qq' = 0$
$mvv' = -kxx'$	$mvv' = -\frac{mg}{l}xx'$	$Lii' = -\frac{1}{C}qq'$
$x' = v$	$x' = v$	$q' = i$
$mv' = -kx$	$mv' = -\frac{mg}{l}x$	$Li' = -\frac{1}{C}q$
$v' = -\frac{k}{m}x$	$v' = -\frac{g}{l}x$	$i' = -\frac{1}{LC}q$

$v' = x''$	$v' = x''$	$i' = q''$
$x'' = -\frac{k}{m}x$	$x'' = -\frac{g}{l}x$	$q'' = -\frac{i}{LC}q$
$let\ \frac{k}{m} = \omega^2$	$let\ \frac{g}{l} = \omega^2$	$let\ \frac{1}{LC} = \omega^2$
$\boldsymbol{x'' = -\omega^2 x}$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$	$\boldsymbol{x'' = -\omega^2 x}$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$	$\boldsymbol{q'' = -\omega^2 q}$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$

It is obvious that various oscillatory occurrences are described with the equations of the same type and have some constant component in common, so this feature can be used for developing the forecast models for ES behavior.

It is also reasonable to consider various types of energy domains to check the MPES forecast function base for the common link, see Figure 4.7.

Every energy domain is characterized by two physical values of the first and second kind, its product is a power always. If we consider an electric domain, they are the electric current and voltage respectively. These pair physical values, in each of the energy domain, are connected to each other by the Ohm's law in the respective formulation.

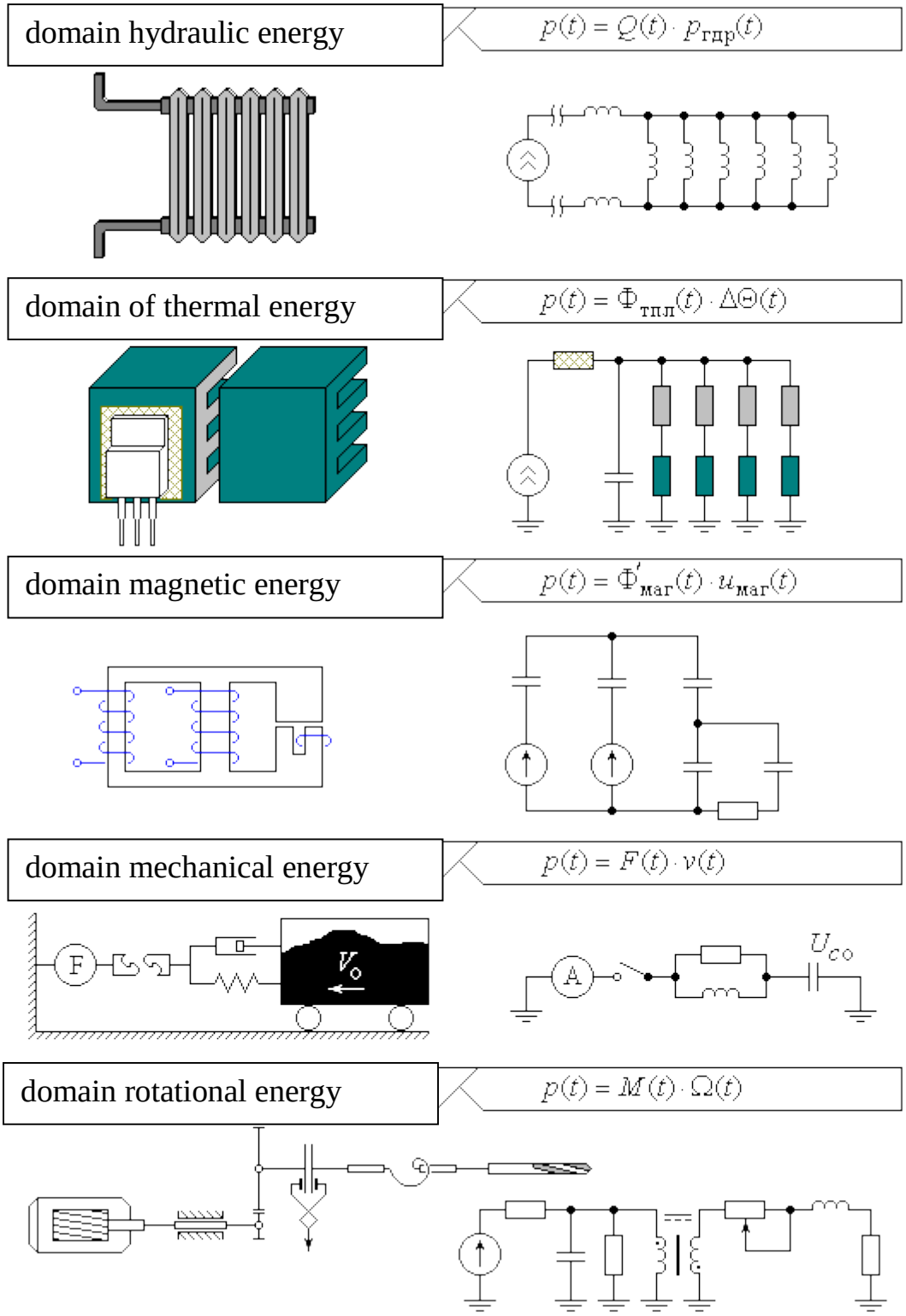


Figure 3.7 Diagram of various energy domain types

Consequently, the electrical, magnetic, thermal, hydraulic, acoustic, mechanical, rotating and other resistances are the energy consumers in each of the energy domains, see Figure 4.8.

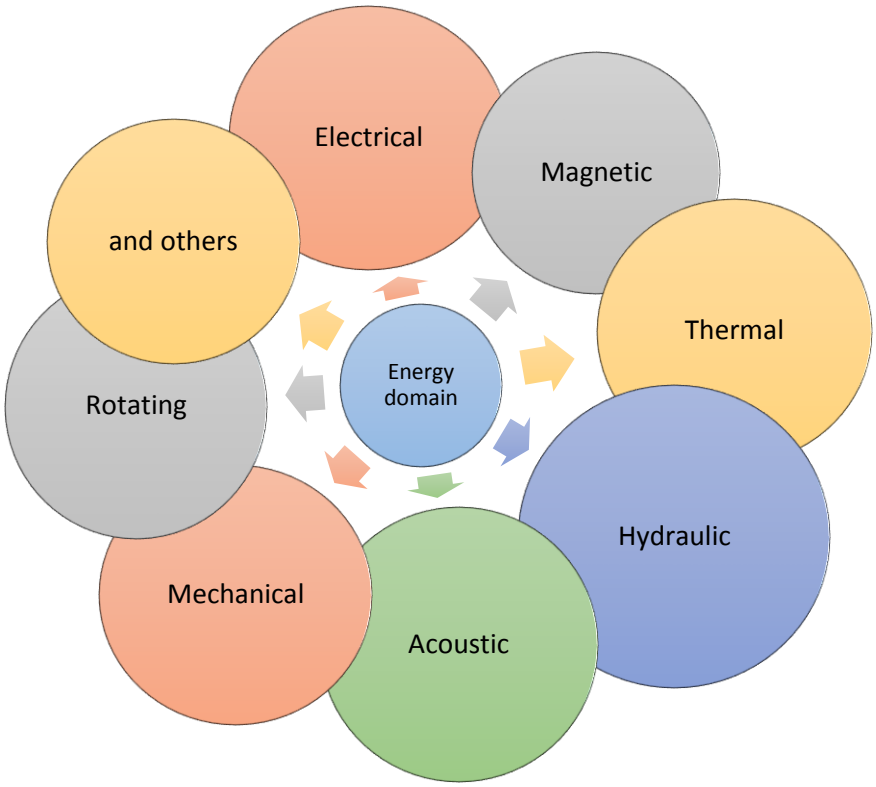
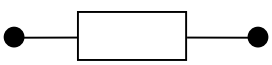
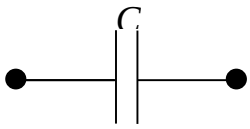


Figure 4.8 Types of resistances that consume the domain energy respectively

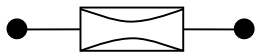
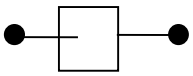
In all the cases, it is a simple physical device subdivided into three classes acording to the operating principle (*R*, *L*, *C*). We can say that the Ohm's law has 21 formulations for seven above mentioned energy domains. The Ohm's law formulations can be written in three different ways (see Table 4.5).

Ohm's law formulations for seven energy domains

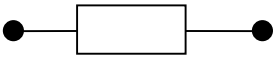
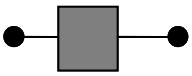
Table 4.5

CLASS OF SIMPLE DEVICES HAVING R OPERATING PRINCIPLE	CLASS OF SIMPLE DEVICES HAVING C OPERATING PRINCIPLE	CLASS OF SIMPLE DEVICES HAVING L OPERATING PRINCIPLE
		
$i(t) = \frac{u(t)}{R}$	$i(t) = C\frac{du(t)}{dt}$	$u(t) = \frac{1}{L}\int u(t)dt$
$u(t) = Ri(t)$	$u(t) = \frac{1}{C}\int i(t)dt$	$u(t) = L\frac{di(t)}{dt}$

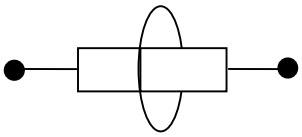
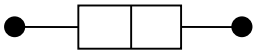
Hydraulic elements

		
$Q(t) = \frac{p(t)}{R}$	$Q(t) = C\frac{dp(t)}{dt}$	$Q(t) = \frac{1}{L}\int p(t)dt$
$p(t) = RQ(t)$	$p(t) = \frac{1}{C}\int Q(t)dt$	$p(t) = L\frac{dQ(t)}{dt}$

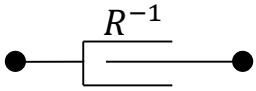
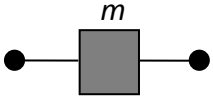
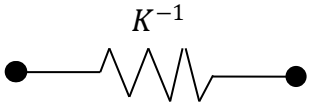
Thermal elements

		
$\Phi(t) = \frac{\Delta\ominus(t)}{R}$	$\Phi(t) = C\frac{d\Delta\ominus(t)}{dt}$	$\Phi(t) = \frac{1}{L}\int \Delta\ominus(t)dt$
$\Delta\ominus(\mathrm{t}) = \mathrm{R}\Phi(\mathrm{t})$	$\Delta\ominus(t) = \frac{1}{C}\int \Phi(t)dt$	$\Delta\ominus(t) = L\frac{d\Phi(t)}{dt}$

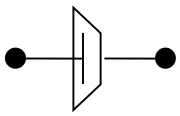
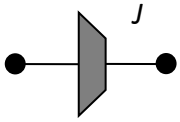
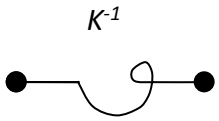
Magnetic elements

		
$\Phi'(t) = \frac{u(t)}{R}$	$\Phi'(t) = C\frac{du(t)}{dt}$	$\Phi'(t) = \frac{1}{L}\int u(t)dt$
$u(t) = R\Phi'(t)$	$u(t) = \frac{1}{C}\int \Phi'(t)dt$	$u(t) = L\frac{d\Phi'(t)}{dt}$

Mechanical elements

		
$F(t) = \frac{v(t)}{R^{-1}}$	$F(t) = m\frac{dv(t)}{dt}$	$F(t) = \frac{1}{K^{-1}}\int v(t)dt$
$v(t) = R^{-1}F(t)$	$v(t) = \frac{1}{m}\int F(t)dt$	$v(t) = K^{-1}\frac{dF(t)}{dt}$

Rotating elements

		
$M(t) = \frac{\Omega(t)}{\mathrm{R}^{-1}}$	$M(t) = J\frac{d\Omega(t)}{dt}$	$M(t) = \frac{1}{K^{-1}}\int \Omega(t)dt$
$\Omega(t) = R^{-1}M(t)$	$\Omega(t) = \frac{1}{J}\int M(t)dt$	$\Omega(t) = K^{-1}\frac{dM(t)}{dt}$

Three Ohm's law notations determine three formalized primitives that are the passive elements of undirected graphs, i.e. the models of energy consumers. There exist their own symbolic graphical notations in each of the energy domains for them, but the mathematical nature of the respective library items is unchanged.

Nine formalized primitives of undirected graphs – energy consumers, sources, junctions and potential ground-wire – allow developing the comprehensive library of physical devices elements with any scale and detail levels.

4.3 Analytical representation of a forecast model as a class of several functions

If the analysis of the source experimental findings shows that *y(x)* dependence describing the source test data is not a monotone increasing or monotone decreasing one but contains a set of relative minimums and maximums, then it may be difficult to find not too cumbersome single empiric formula.

In such cases it is reasonable to represent a required dependence as a class of several functions, e.g. as Figure 4.9 shows.

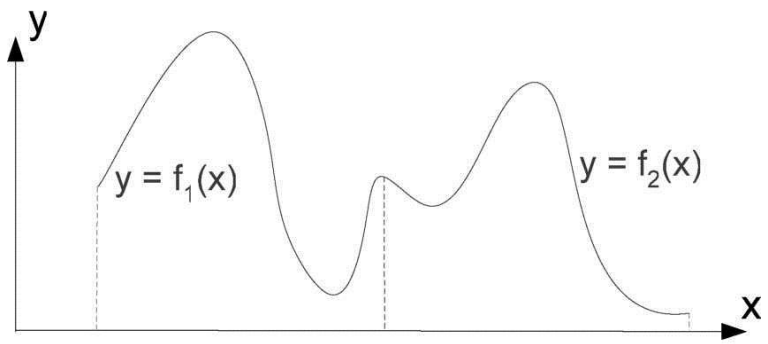


Figure 4.9 Splitting the argument span into isotropic intervals

Doing this requires splitting the source data span into several isotropic intervals and defining its own function for each of the intervals. In such a case, we have the problem of finding a compromise between choosing the minimum number of split intervals, ensuring the required accuracy and reaching the computational "simplicity" of the obtained functions.

One of the simplest options of span splitting is the extreme points splitting, see Figure 4.10. In such a case, as the source data is given as a finite discrete data array, the required function minimums and maximums can be defined by sequential comparison of the values of the array neighboring elements.

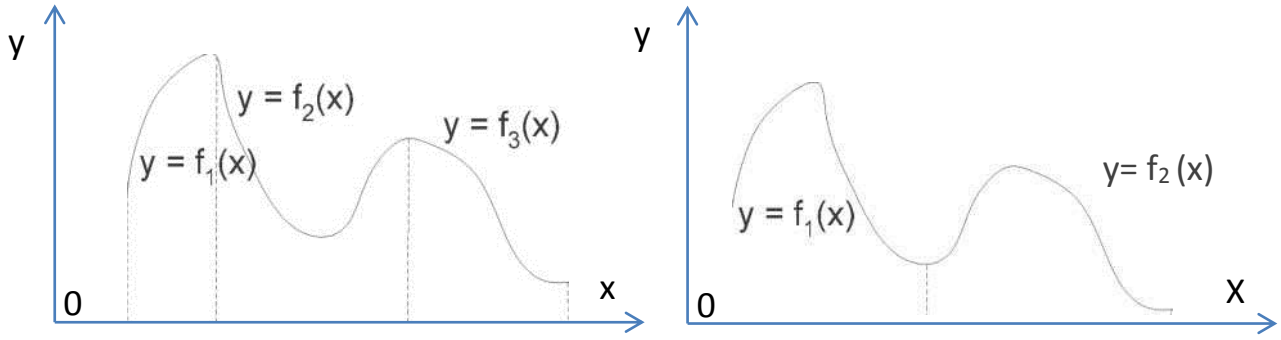


Figure 4.10 Splitting by the extreme points

The other possible option is splitting by the flex points, see Figure 4.11. If we deal with an analytical function, we may find the flex points by computing zeros of the second function derivative. In our case, if we know only the discrete function values, we may find the approximate flex points by computing the increment of a function in its neighbor points.

The source test data often have a lot of extreme and flex points. However, *the number of splitting intervals should be minimal*. Consequently, there is a problem of choosing several points out of the whole set of special points for detecting the splitting intervals borders

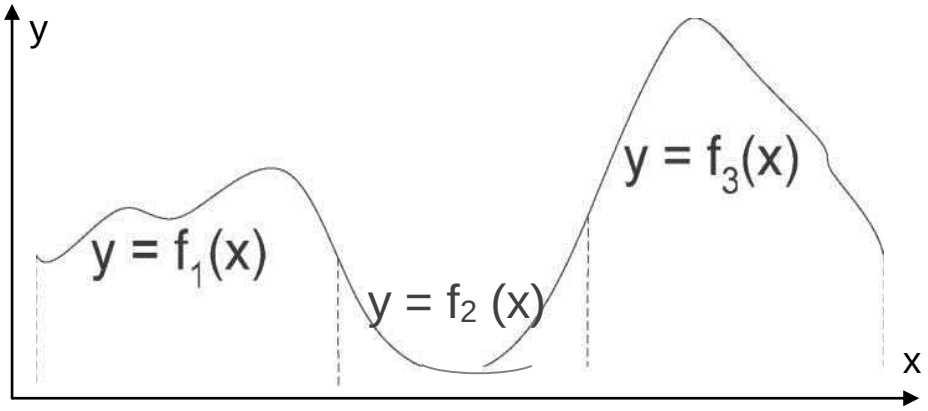


Figure 4.11 Splitting by the flex points

We may select the most standing out extreme points applying the first-order function derivative calculated approximately with the help of the calculus. For example, the derivative value may be calculated with the formula:

$$f'(x) = \frac{1}{2}\left(\frac{y_{i+1}-y_i}{x_{i+1}-x_i} + \frac{y_i-y_{i-1}}{x_i-x_{i-1}}\right). \quad (4.16)$$

In some cases, we may *choose the points of the span interval splitting according to the physical peculiarities of the investigated phenomenon*. For example, if we develop the analytical dependence of the atmosphere transmission factor from the wavelength, it is reasonable to split the spectral range into the intervals in such a way that each interval corresponds to the known transmission band.

4.4 Method of developing the forecast models having various degree of detail

Let us consider the development of the analytical forecast model basing on the test data concerning the values of some variable *y* depending on the value of the other variable *x* that have been obtained at various values of *w, v, g* and *q* factors. What is more, by factors we mean both the influencing variables and some environmental

conditions. For example, if we investigate the atmosphere transmission phenomenon, the environmental conditions are the meteorological optical range, climate and geographical setting.

In accordance with the principles of analytical model development considered in Chapter 1, at stage one we need to set the aims and application conditions of the required forecast models, to formulate the requirements specified to it.

Let us suppose that *w, v* and g factors may be measured at a definite moment of the investigated phenomenon evaluation, and the *q* factor value is unknown at the moment. It means that to develop the most accurate model on the basis of the source data we should account for the influence of *w, v* and g factors in its structure. At the same time, the model should be invariant to the *q* factor.

Consequently, as a source data we have a set of discrete data concerning the $y\ (x, w_i, v_j, g_r, q_s)$ dependence, where $i = 1...m$, $v...l$, $g...k$, $1...p$. Here *m, l, k, p* is the number of options for various values of *w, v, g, q* factors respectively, for which the experiments have been conducted. We need to obtain an analytical dependence for the source data in the form of $y = f\ (x, w_i, v_j, g_r)$.

Let us apply "build-up" approach to a modelling. It consists in "adding" the influence of separate factors gradually when developing an analytical forecast model. Figure 4.12 shows the generalized algorithm of obtaining the analytical model

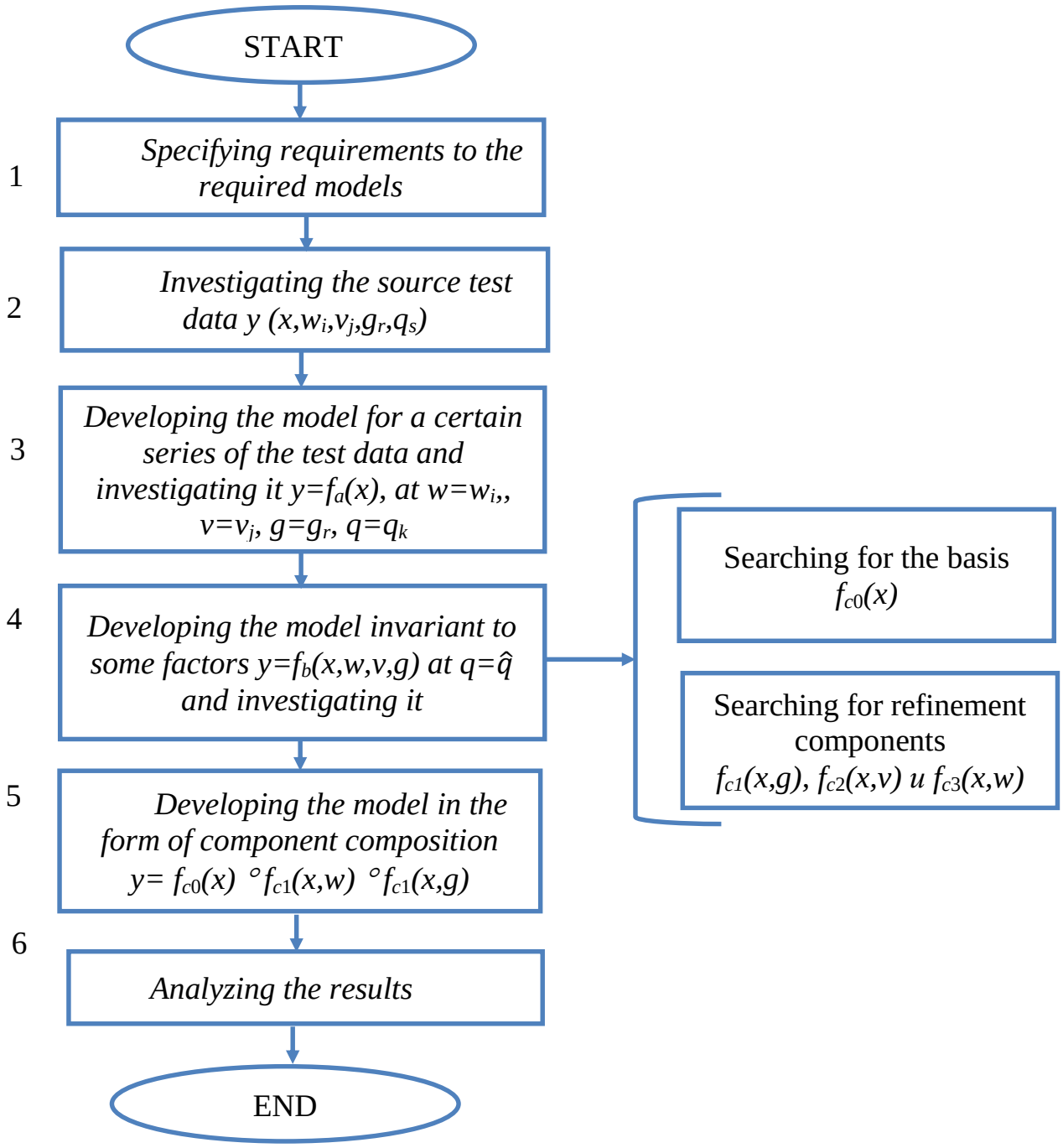


Figure 4.12 Algorithm for analytical model development

At stage one we specify the requirements to the required models.

At stage two we perform the data primary processing and investigation. Here we discretize and normalize the source data, if necessary. We distinguish the domain set and the set of admissible values on account of both the data itself and the known physical laws governing the investigated phenomenon. We also specify the peculiar

features of the required dependence: monotonous and constant-signs intervals, discontinuity and flex points, etc.

At stage three we develop a model for a certain data series, i.e. we draw an analytical dependence $y = f_a(x)$ for some definite factor values $w = w_i$, $v = v_j$, $g = g_r$, $q = q_s$. To develop a model, we apply an algorithm shown at Figure 4.12 step-by step. The investigation of the model developed in such a way may indicate some specific operational characteristics important for the given process. Anyhow, we need to evaluate how successful the chosen points of dividing into the isotropic intervals and the tried classes of functions turn out to be for the investigated data.

Then at stage four we offer to develop a model invariant to one or several factors – depending on the requirements specified. In our case under investigation, at stage four we develop an analytical model $y = f_b(x, w, v, g)$ that is invariant to q factor, which values remain unknown at applying the required model for the forecasting. That is why $y = f_b(x, w, v, g)$ model is developed for some particular or somehow averaged value of factor $q=\tilde{q}$. Factors w, v are g may be accounted in the form of various indices values in the model structure at this stage. Initial suppositions and modeling aims influence on the method of obtaining the invariant model, as far as the results of test data investigation. For example, there are the following options for "excluding" a factor out of the model:

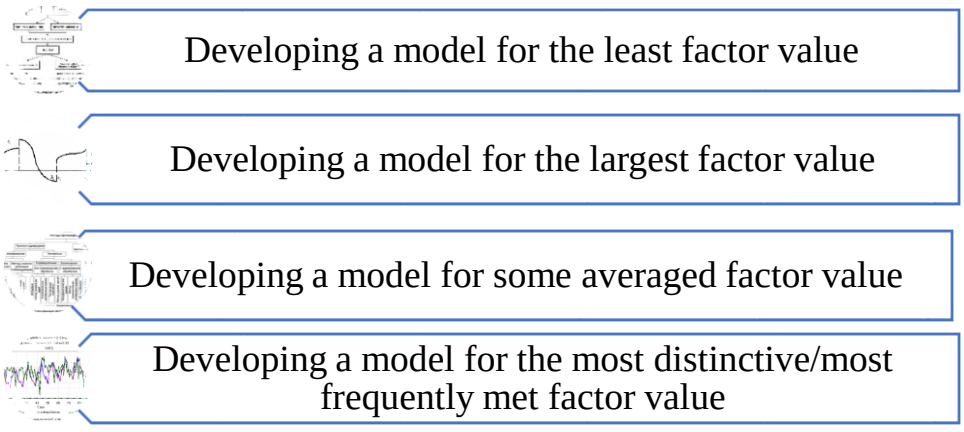


Figure 4.13 Options for excluding factors out of the model

At stage five we develop a model in the form of the component composition. We may divide it into the substages of searching for the basis and for refinement components.

The analytical model structure in the form of the component composition looks as follows:

$$y = f_{c0}(x) \circ f_{c1}(x, s) \circ f_{c2}(x, r). \quad (4.16)$$

Here $f_{c0}(x)$ is a basis making for a model "base". In an ideal case, *the basis reveals the investigated phenomenon, describes its most distinctive behavior*; that is called a trend or a systematic component at considering temporal serieses.

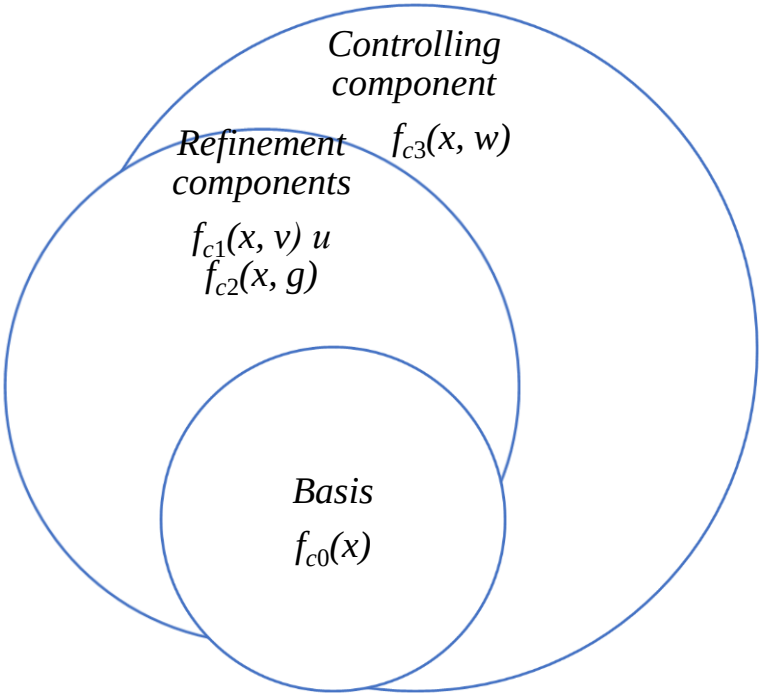


Figure 4.14 Forecast function structure

While developing an empiric forecast model for some of the purposes, the basis may also be drawn out of the worst suppositions concerning the physical phenomenon development.

$f_{c1}(x, v)$, $f_{c2}(x, g)$ and $f_{c3}(x, w)$ are the refinement components and introduced to the model for accounting the influence of v, g and w factors on the investigated phenomenon respectively.

Unfortunately, it is not always possible to find a component composition ensuring the specified accuracy. In such case, we may turn our attention to the invariant model

from stage 4 and on accounting for the influence of *v*, *g* and *w* factors with respect to different indices in empiric correlations.

At stage five we analyze the results.

4.5 Obtaining the forecast model basis

The model basis allows obtaining a forecast of the lowest detail level, i.e. estimating crudely the technical process state before receiving some additional information on its internal and external parameters.

Depending on the modelling aims, initially specified limitations and model application conditions, different variations of the source test data may be the core of the basis.

If there is enough information concerning the most distinctive existence conditions of the investigated technical process, it is reasonable to use the observations received in that same or the most closely resembling conditions for the basis development.

The other option for basis development is to use the averaged data from the conducted experiments. However, here there is a problem of choosing the averaging method – computing the arithmetic mean from all of the data, applying some integrated index and so on.

For some of the modelling purposes (e.g. forecasting the abnormal situations), minimums or, on the contrary, maximums of the investigated process parameters that characterize its worst or conversely the most favorable state may be taken as the basis core.

If we decide to develop the basis as some averaged model describing the general behavior of the investigated technical process, then it is possible to apply the smoothing algorithms for its development. In this case, the smoothing may be done by using the splines [10] or Chebyshev polynomials [3], Fourier series [53] or wavelets [54], median [52] or exponential smoothing methods [49], with different filters (linear or Savitzky-Golay filters [58]), etc.

4.6 Accounting for additional factors

Figure 4.15 shows the application of the fast Fourier transformation for basis obtaining at the point of extrapolating the wind velocity data.

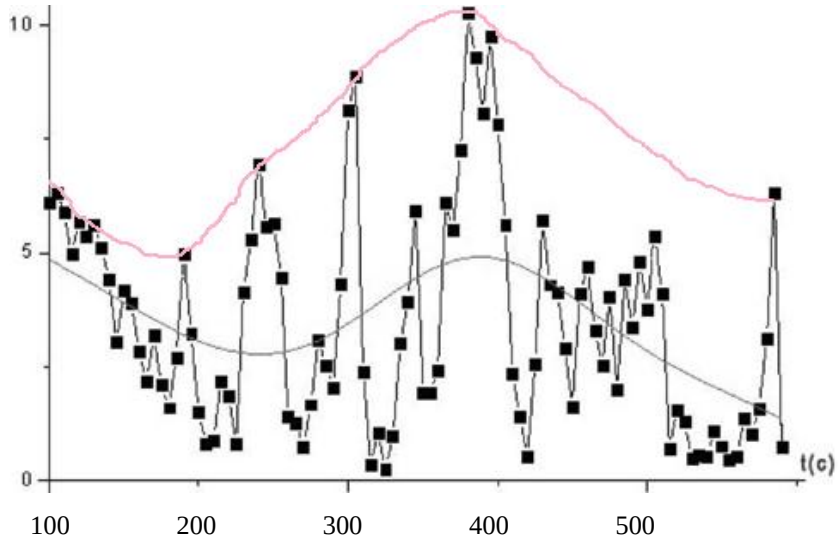


Figure 4.15 Data smoothing with the fast Fourier transformation

We introduce the additional component into the physical phenomenon model for accounting the influence of some additional conditions. Figure 4.15 shows the graphical representation of the analytical model of wind velocity change in the form of the basis: “the primal trend” and the model with the additional component introduced for accounting the maximums – “maximum estimate”.

Similar to the basis, developing the additional refinement components is also a complex problem, moreover, it directly depends on how the basis has been obtained.

It should be noted, that the introduction of any of the additional components may be motivated by the following reasons:

- *The necessity of accounting for some factor in the model that influence on the investigated object behavior;*
- *The necessity of applying the model under the conditions differing from those under which the basis has been obtained*.

One of the methods of additional component development is to obtain an analytical representation of the data (with the application of the generalized algorithm considered in Chapter 2) obtained with the elementwise subtraction of the basis values

from the test data definite values (obtained for those conditions for account of which we introduce the additional component).

The other method is to investigate the interconnections between the source test data obtained under different conditions.

Besides, in some cases the known physical laws may determine the additional component type.

REFERENCE LIST

1. Айвазян С.А., Енюков И.С., Мешалкин Л.Д. Прикладная статистика. Исследование зависимостей: справочное издание - М. : Финансы и статистика, 1985. -487 с.
2. Айвазян С.А., Енюков И.С., Мешалкин Л.Д. Прикладная статистика: Основы моделирования и первичная обработка данных. Справочное издание. - М. : Финансы и статистика, 1983. - 471 с.
3. Айфичер Э., Джервис Б. Цифровая обработка сигналов: практический подход. Второе издание. Пер. с англ. - М. : Изд. Дом "Вильямс", 2004–992 с.
4. Барлоу Р., Прошан Ф. Математическая теория надежности. Пер. с англ. - М. : "Советское радио", 1969. – 488 с.
5. Браверман Э.М., Мучник И.Б. Структурные методы обработки эмпирических данных. - М. : Наука, 1983. - 464 с.
6. Брайт Дж. Научно-техническое прогнозирование для промышленности и правительственных учреждений, 1968 г.
7. Брайт Дж. Руководство по научно-техническому прогнозированию, 1973 г.
8. Ван Трис Г. Теория обнаружения оценок и модуляции–М. : Советское радио, 1972. - 744 с.
9. Вапник В.Н. Восстановление зависимостей по эмпирическим данным. - М. : Наука, 1979. - 448 с.
10. Василенко В.А. Сплайн-функции: теория, алгоритмы, программы. - Новосибирск : Наука, 1983. - 224 с.
11. Васильков Ю.В., Василькова Н.Н. Компьютерные технологии вычислений в математическом моделировании: учеб, пособие. - М. : Финансы и статистика, 2001. - 256 с.
12. Демидович Б.П., Шувалова Э.З., Марон И.А. Численные методы анализа: Приближение функций, дифференциальные и интегральные уравнения: (под ред. Демидовича Б.П.) Изд. 3-е, перераб. - М. : Наука, 1967. - 368 с.

13. Демин, А.В., Копорский Н.С. Имитационное моделирование информационно-измерительных и управляющих систем // Монография. – СПб. : Университет ИТМО, 2007. – 138 с.
14. Демин А.В., Дмитриева С.П. Разработка алгоритмов построения прогнозных многопараметрических моделей физических явлений // III Труды Конгресса по интеллектуальным системам и информационным технологиям "IS&IT' 14 ". Научное издание в 4-х томах. - М. : Физматлит, 2014. - Т. 3. - С. 289 - 294.
15. Демин А.В., Моисеева М.И. Оценка достоверности представления экспериментальных данных аналитической зависимостью // II Труды Конгресса по интеллектуальным системам и информационным технологиям "IS&IT' 11 ". Научное издание в 4-х томах. - М. : Физматлит, 2011. - Т. 1. - С. 414-416.
16. Дирак П. А. М. Основы квантовой механики / Пер. с англ. — М., 1932 (есть много переизданий).
17. Дмитриева С.П. Алгоритм построения аналитических прогнозных функций на основе экспериментальных данных // Материалы III Международная научная конференция ИТМО.
18. Дмитриева С.П. Алгоритм построения прогнозных функций. // Материалы X Международная научная конференция «Интеграция науки и практики как механизм эффективного развития современного общества», 28.12.2013, Россия. http://www.gosnauka.ru/.
19. Дмитриева С.П. Аналитический обзор: методология компьютерного моделирования. // Материалы IX Международная научная конференция «Тенденции и перспективы развития современного научного знания», 27.12.2013, Россия. http://teoretika.com/.
20. Дмитриева С.П. Аналитический обзор: основные принципы моделирования процессов. // Материалы X Международная научная конференция «Теоретические и практические аспекты развития современной науки», 23.12.2013, Россия. http://www.ruskonf.ru/.

21. Дрейпер Н., Смит Г. Прикладной регрессионный анализ: в 2-х кн. 1/ Пер. с англ.-2-е изд., перераб. идоп. - М. : Финансы и статистика, 1986. - 366 с.
22. Зайцев В. Ф. Математические модели в точных и гуманитарных науках. - СПб. : 000 "Книжный Дом" , 2006. -112 с.
23. Замятина О.М. Моделирование систем. Учебное пособ. для Вузов. - Томск : ТПУ, 2009. - 204 с.
24. Карташов В.Я., Штефан В.В. Исследование статистических зависимостей методом обратных разделенных разностей / Деп. статья N 1105 - В92. - М.: ВИF-IIиТИ, 1992.- 5 с.
25. Кобзарь А.И. Прикладная математическая статистика. Для инженеров и научных работников. - М. : Физматлит, 2006. - 816 с.
26. Копаев Б. В. Использование условно-относительных разностей при определении коэффициентов полинома методом наименьших квадратов // Надежность и контроль качества. - 1991. - N12. - С. 3-12.
27. Копаев Б.В. К вопросу использования относительных разностей в методе наименьших квадратов // Геодезия и картография. - 1991. - N6. - С. 9-13.
28. Краснопевцев Е. А. Математические методы физики. Избранные вопросы.
29. Кудрявцев Л. Д. Краткий курс математического анализа. — Том 2. — ISBN 5-9221-0185-4.
30. Львовский Е.Н. Статистические методы построения эмпирических формул. Учебное пособ. для Вузов. - 2-е изд., - М. : Высш. школа, 1988. - 239 с.
31. Любушин А.А. Анализ данных систем геофизического и экологического мониторинга. - М. : Наука, 2007. - 228 с.
32. Моисеева М.И. Критерии выбора функции экстраполяции данных для оценки нестационарных случайных процессов //Труды Конгресса по интеллектуальным системам и информационным технологиям "IS&IT'11". Научное издание в 4-х томах. - М. : Физматлит, 2011. - Т. 3. - С. 61-67.
33. Моисеева М.И. Разработка алгоритма аналитического представления экспериментальных данных // Аннотированный сборник научно-исследовательских выпускных квалификационных работ магистров

СП6ГУ ИТМО / Главный редактор д.т.н., проф. В.О. Никифоров. - СПб : СПбГУ ИТМО, 2010. - С. 34-35.

34. Моисеева М.И. Эмпирический подход в математическом моделировании физических явлений // Сборник научных трудов Sworld. Материалы международной научно-практической конференции «Современные проблемы и пути их решения в науке, транспорте, производстве и образовании 2011». -Вьшуск 4. Том 4. - Одесса: Черноморье,2011. - С.27-29.
35. Мудров В.Ж., Кушко В.Л. Метод наименьших модулей. – М. : Знание, 1971. - 31 с.
36. Мышкис А.Д. Элементы теории математических моделей. - 3-е изд., испр. - М. : Дом Книга, 2007. - 192 с.
37. Носач В.В. Решение задач аппроксимации с помощью персональных компьютеров. - М.: Микап, 1994. - 382 с.
38. Очин Е.Ф. Вычислительные системы и обработка изображений. - Л. : Энергоатомиздат, 1989. 136 с.
39. Рабинер Л., Гоулд Б. Теория и применение цифровой обработки сигналов. - М.: Мир, 1978. - 545 с.
40. Розенберг Г.С., Шитиков В.К., Брусиловский П.М. Экологическое прогнозирование (Функциональные предикторы временных рядов). - Тольяти, 1994. - 182 с.
41. Самарский А.А., Михайлов А.П. Математическое моделирование: Идеи. Методы. Примеры. 2-ое изд., испр. - М. : Физматлит, 2001. -320 с.
42. Ситрон М., Барток Б. Переоценка технологии в динамической среде, 1974 г.
43. Тейлор Дж.. Введение в теорию ошибок. Пер. сангл. -М. : Мир, 1985. -272 с.
44. Цыцулин А.К. Телевидение и космос: учеб. пособие. - СПб. : СПбГЭТУ «ЛЭТИ». 2014. – 237 с.
45. Чураков Е.П. Математические методы обработки экспериментальных данных в экономике: учеб. пособие. -М. : Финансы и статистика. 2004.-240с.
46. Ярская В.Н. Методология диссертационного исследования // Методическое пособие. - Саратов : СГТУ, 2011. - 170 с.

47. Bassett E.E., Bremner J.M., Morgan B.J.T. Statistics: Problems and Solutions. - World Scientific, 2000. - 227 p.
48. Brown R.G. Smoothing, forecasting and prediction of discrete time series. - Courier Dover Publications, 2004. - 480 p.
49. Clive L.D. Principles of mathematical modeling. -- Elsevier Academic Press, 2004. - 303 p.
50. Dmitrieva S.P. Development of scientific forecasting. / Materials of IV international scientific conference «Science, Technology and Higher Education», 30.01.2014, Canada. http://science-canada.com/.
51. Dmitrieva S.P. Development of scientific forecasting: building predictive models. / Materials of V international scientific conference «Science and Education», 27-28.02.2014, Germany. http://www.euscience.de/.
52. Johnson R.A., Bhattacharyya G.K. Statistics: Principles and Methods. -- John Wiley & Sons, 2009. - 704 p.
53. Katkovnik V., Egiazarian K., Astola J. Local approximation techniques in signal and image processing. - SPIE Press, 2006. - 553 p.
54. Kauppinen J., Partanen J. Fourier transforms in spectroscopy. - John Wiley & Sons, 2011. -271 p.
55. Kobayashi M. Wavelets and their applications: case studies. — SIAM, 1998. — 142 p.
56. Livingstone D.A practical guide to scientific data analysis. -- John Wiley &Sons, 2010. - 358 p.
57. Motulsky H.J., Christopoulos A. Fitting models to biological data using linear and nonlinear regression. A practical guide to curve fitting. - GraphPad Software Inc., San Diego CA, -2003. - 351 p.
58. Oldham K.B., Myland J.C., Spanier J. An Atlas of Functions: With Equator, the Atlas Function Calculator. - NY: Springer, 2008. - 748 p.
59. Rosenthal J., Gilliam D.S. Mathematical systems theory in biology, communications, computation and finance. - Springer, 2003. - 504 p.

60. Thompson W.J. Atlas for computing mathematical functions: an illustrated guide for practitioners with programs in Fortran 90 and Mathematica. - Wiley, 1997. - 888 p.
61. Velten K. Mathematical Modeling and Simulation: Introduction for Scientists and Engineers. - John Wiley & Sons, 2009. - 362 p.

APPENDIX 1

OPTIMIZATION OF THE OPTICAL BEAM CONTROL ALGORITHM WITH THE SIMULATION MODELLING METHOD

Along with the analytical methods of solving the problems connected with the search of the implemented system control algorithm, the simulation modelling methods may be applied. The specific character of the simulation modelling is an opportunity to estimate the whole system behavior by simulation results of its subsystems in accordance with its simulation models, the aggregate of which is simpler than the model of the whole system. However, the simulation fidelity is mostly defined by the result of the system decomposition and the correctness of the simulation modelling algorithm implementation.

Let us consider the application of simulation modelling method for the search of the optimum algorithm of the optical beam control in two coordinates with a couple of optical wedges. In general terms, the analytical correlations describing the system operation include the values of refraction indices, refracting wedge angles and its rotation angles. Naturally, it is almost impossible to search for the optimum control algorithm with simultaneous consideration of the image quality (chromatic aberration). In that context it is reasonable to decompose the system into two subsystems: a subsystem ensuring the beam movement and a subsystem ensuring the image quality. Then we may divide the search of the optimum algorithm into three stages:

Searching for the algorithm of the wedge rotation at preliminary chosen values of refracting angles and material properties.

Searching for the values of refracting angles and refraction indices ensuring the required image quality.

Correcting the wedge rotation algorithm with account of the second stage results.

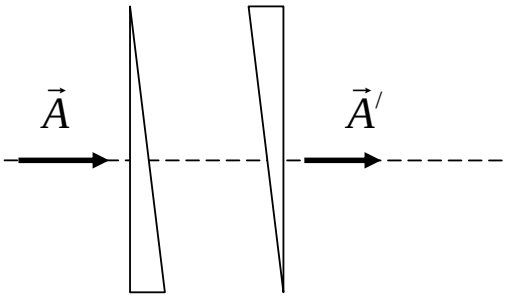
The first stage is the most difficult one, let us consider it.

Refraction law in the vectorial form:

$$\begin{cases} \vec{A}_m = \dfrac{n_{m-1}}{n_m}(\vec{A}_{m-1} + \Gamma_m \vec{N}_m) \\ \Gamma_m = \cos\alpha_m - \dfrac{n_m}{n_{m-1}}\sqrt{1-(\dfrac{n_{m-1}}{n_m})^2(1-\cos^2\alpha_m)} \\ \cos\alpha_m = -\vec{A}_{m-1}\vec{N}_m \end{cases}$$

where $\vec{A}_m$ is the vector of a beam refracted from "m-1" to "m" medium having n_{m-1} and n_m refraction indices respectively; α_m - the incidence angle of $\vec{A}_{m-1}$ beam to "*m*" surface having $\vec{N}_m$ normal line.

Let:
$$\begin{cases} \vec{A} = \vec{j}, \vec{N}_2 = \vec{N}_3 = -\vec{j} \\ \vec{N}_1 = \vec{i} \times \sin\sigma_1 \sin\phi_1 - \vec{j} \times \cos\sigma_1 + \vec{k} \times \sin\sigma_1 \cos\phi_1 \\ \vec{N}_2 = \vec{i} \times \sin\sigma_2 \sin\phi_2 - \vec{j} \times \cos\sigma_2 + \vec{k} \times \sin\sigma_2 \cos\phi_2 \end{cases}$$



$$\begin{cases} \vec{A}' = (\Gamma_1 \sin\sigma_1 \sin\phi_1 + n_2\Gamma_4 \sin\sigma_2 \sin\phi_2) \times \vec{i} + (1 - \Gamma_1 \cos\sigma_1 - \\ - n_2\Gamma_4 \cos\sigma_2) \times \vec{j} + (\Gamma_1 \sin\sigma_1 \cos\phi_1 + n_2\Gamma_4 \sin\sigma_2 \cos\phi_2) \times \vec{k} \\ \Gamma_1 = \cos\sigma_1 - \sqrt{n_1^2 - \sin^2\sigma_1} \\ n_2\Gamma_4 = -(\Gamma_1 \sin\sigma_1 \sin\sigma_2 \cos(\phi_1 - \phi_2) - (1 - \Gamma_1 \cos\sigma_1)\cos\sigma_2 - \\ - \sqrt{1 - n_2^2(1 - (\Gamma_1 \sin\sigma_1 \sin\sigma_2 \cos(\phi_1 - \phi_2) - (1 - \Gamma_1 \cos\sigma_1)\cos\sigma_2)^2)} \end{cases}$$

sin 20 - 20π/1800 ≈ 1,06×10-5; sin 50 - 50π/1800 ≈ - 6,65×10-5 ⇒ at σ ≤ 50 possible to

$$\begin{cases} \vec{A}' = (\Gamma_1\sigma_1 \sin\phi_1 + n_2\Gamma_4\sigma_2 \sin\phi_2)\vec{i} + (1-\Gamma_1\sqrt{1-\sigma_1^2} - \\ -n_2\Gamma_4\sqrt{1-\sigma_2^2})\vec{j} + (\Gamma_1\sigma_1 \cos\phi_1 + n_2\Gamma_4\sigma_2 \cos\phi_2)\vec{k} \\ \Gamma_1 = \sqrt{1-\sigma_1^2} - \sqrt{n_1^2-\sigma_1^2} \\ n_2\Gamma_4 = -(\Gamma_1\sigma_1\sigma_2 \cos(\phi_1-\phi_2) - (1-\Gamma_1\sqrt{1-\sigma_1^2})\sqrt{1-\sigma_2^2} - \\ -\sqrt{1-n_2^2(1-(\Gamma_1\sigma_1\sigma_2 \cos(\phi_1-\phi_2) - (1-\Gamma_1\sqrt{1-\sigma_1^2})\sqrt{1-\sigma_2^2})^2)} \end{cases}$$

in XOZ projection plane at the distance L in linear measure

$$\begin{cases} x_L^{/} = (\Gamma_1\sigma_1 \sin\phi_1 + n_2\Gamma_4\sigma_2 \sin\phi_2)L \\ z_L^{/} = (\Gamma_1\sigma_1 \cos\phi_1 + n_2\Gamma_4\sigma_2 \cos\phi_2)L \\ \Gamma_1 = \sqrt{1-\sigma_1^2} - \sqrt{n_1^2-\sigma_1^2} \\ n_2\Gamma_4 = -(\Gamma_1\sigma_1\sigma_2 \cos(\phi_1-\phi_2) - (1-\Gamma_1\sqrt{1-\sigma_1^2})\sqrt{1-\sigma_2^2} - \\ -\sqrt{1-n_2^2(1-(\Gamma_1\sigma_1\sigma_2 \cos(\phi_1-\phi_2) - (1-\Gamma_1\sqrt{1-\sigma_1^2})\sqrt{1-\sigma_2^2})^2)} \end{cases}$$

and angular measure respectively

$$\begin{cases} Y_X = \Gamma_1\sigma_1 \sin\phi_1 + n_2\Gamma_4\sigma_2 \sin\phi_2 \\ Y_Z = \Gamma_1\sigma_1 \cos\phi_1 + n_2\Gamma_4\sigma_2 \cos\phi_2 \\ \Gamma_1 = \sqrt{1-\sigma_1^2} - \sqrt{n_1^2-\sigma_1^2} \\ n_2\Gamma_4 = -(\Gamma_1\sigma_1\sigma_2 \cos(\phi_1-\phi_2) - (1-\Gamma_1\sqrt{1-\sigma_1^2})\sqrt{1-\sigma_2^2} - \\ -\sqrt{1-n_2^2(1-(\Gamma_1\sigma_1\sigma_2 \cos(\phi_1-\phi_2) - (1-\Gamma_1\sqrt{1-\sigma_1^2})\sqrt{1-\sigma_2^2})^2)} \\ \max\{Y\} = \Gamma_1\sigma_1 + (-(\Gamma_1\sigma_1\sigma_2 - (1-\Gamma_1\sqrt{1-\sigma_1^2})\sqrt{1-\sigma_2^2} - \\ -\sqrt{1-n_2^2(1-(\Gamma_1\sigma_1\sigma_2 - (1-\Gamma_1\sqrt{1-\sigma_1^2})\sqrt{1-\sigma_2^2})^2)})\sigma_2 \end{cases}$$

To find the motion trajectory, i.e. the control action for φ_1 and φ_2, under the condition that $|\Gamma_1\sigma_1| = |n_2\Gamma_4\sigma_2| = \Gamma$ and taking into account that $sign(\Gamma_1\sigma_1) = -sign(n_2\Gamma_2\sigma_2)$ at $\Gamma_1\sigma_1 \prec 0, n_2\Gamma_4\sigma_2 \succ 0$, we obtain

$$\begin{cases} Y_X = \Gamma(\sin\phi_2 - \sin\phi_1) \\ Y_Z = \Gamma(\cos\phi_2 - \cos\phi_1) \end{cases}$$

Applying the famous trigonometric transformation, we obtain:

$$\begin{cases} Y_x = 2\Gamma \sin\beta \cos\alpha \\ Y_z = -2\Gamma \sin\alpha \sin\beta \end{cases},$$

where $\alpha = \frac{\varphi_2 + \varphi_1}{2}$, a $\beta = \frac{\varphi_2 - \varphi_1}{2}$. Then, if $\cos\alpha \neq 0$:

$$\begin{cases} \sin\beta = \dfrac{Y_x}{2\Gamma \cos\alpha} \\ Y_z = -2\Gamma \sin\alpha \dfrac{Y_x}{2\Gamma \cos\alpha} \end{cases} \text{ and, at } Y_x \neq 0:$$

$$\begin{cases} \alpha = \operatorname{arctan}(-\dfrac{Y_z}{Y_x}) + \pi k \\ \beta = (-1)^t \arcsin\left[\dfrac{Y_x}{2\Gamma \cos\left[\operatorname{arctan}(-\dfrac{Y_z}{Y_x}) + \pi k \right]} \right] + \pi t \\ \alpha \neq \pi/2 + \pi m \\ Y_x \neq 0 \end{cases}, \text{ where } k,t,n \in Z.$$

It should be noted, that fulfilling the $\cos\alpha \neq 0$ condition is the consequence of fulfilling the $Y_x \neq 0$ condition. Indeed:

$$\cos\alpha \neq 0 \Rightarrow \alpha \neq \frac{\pi}{2} + \pi k, k \in Z \Leftrightarrow \operatorname{arctan}(-\frac{Y_z}{Y_x}) \neq \frac{\pi}{2} + \pi k, k \in Z \Leftarrow Y_x \neq 0.$$

Then we obtain:

$$\begin{cases} \varphi_1 = \arctan(-\dfrac{Y_z}{Y_x}) + \pi k - (-1)^t \arcsin\left[\dfrac{Y_x}{2\Gamma \cos\left[\arctan(-\dfrac{Y_z}{Y_x}) + \pi k \right]} \right] + \pi t \\ \varphi_2 = \arctan(-\dfrac{Y_z}{Y_x}) + (-1)^t \arcsin\left[\dfrac{Y_x}{2\Gamma \cos\left[\arctan(-\dfrac{Y_z}{Y_x}) + \pi k \right]} \right] + \pi t \\ Y_x \neq 0 \end{cases}$$, where

$$k, t \in Z .$$

Now it is necessary to consider the case when $Y_x = 0$:

$$\begin{cases} Y_x = 0 \\ \left[\begin{array}{l} \cos\alpha = 0 \Leftrightarrow \varphi_2 + \varphi_1 \in \pi + 2\pi k, k \in Z \\ \sin\beta = 0 \Leftrightarrow \varphi_2 + \varphi_1 \in 2\pi k, k \in Z \end{array} \right. \end{cases}.$$

In all, we obtain:

$$\left[\begin{array}{l}
\left\{\begin{array}{l}
Y_x \neq 0 \\
\varphi_1 = \arctan(-\dfrac{Y_z}{Y_x}) + \pi k - (-1)^t \arcsin\left[\dfrac{Y_x}{2\Gamma\cos\left[\arctan(-\dfrac{Y_z}{Y_x}) + \pi k\right]}\right] + \pi t \\
\varphi_2 = \arctan(-\dfrac{Y_z}{Y_x}) + (-1)^t \arcsin\left[\dfrac{Y_x}{2\Gamma\cos\left[\arctan(-\dfrac{Y_z}{Y_x}) + \pi k\right]}\right] + \pi t
\end{array}\right. \\
\left\{\begin{array}{l}
Y_x = 0 \\
\left[\begin{array}{l}
\cos\alpha = 0 \Leftrightarrow \varphi_2 + \varphi_1 \in \pi + 2\pi k \\
\sin\beta = 0 \Leftrightarrow \varphi_2 + \varphi_1 \in 2\pi k
\end{array}\right.
\end{array}\right.
\end{array}\right.$$

where $k, t \in Z$.

It is evident that an analytical solution of the system is very complicated. Besides, it is impossible to use continuous functions when dealing with real-world controllers that regulate the optical guidance systems. In that context, even having an analytical solution, we perform the results sampling aimed at obtaining some value list of angular displacements of optical wedges depending on the optical beam deviation at the stage of programming the controllers. Consequently, we may omit the stage of analytical solution development.

Mathematically we may express the problem as searching for the unknown rotation angles of the pair of optical wedges with the preliminary system simulation at the specified output deviation angles.

To solve the problem, let us conduct several experiments on a computer. We make a table to put down all the experiments results. We choose a sample spacing and try to cover all the acceptable region of φ_1, φ_2 with a discrete network, with the

nodes $\varphi_{1,}^{i}\varphi_{2}^{j}$ of which we may compute $Y_x(\varphi_{1,}^{i}\varphi_{2}^{j})$ and $Y_z(\varphi_{1,}^{i}\varphi_{2}^{j})$, and put it down into the Table.

It should be noted, that we need to work with the first quarter of values Y_X, Y_Z only, because:

$$Y_x(\varphi_1,\varphi_2) = -Y_x(\varphi_2,\varphi_1)$$
$$Y_z(\varphi_1,\varphi_2) = -Y_z(\varphi_2,\varphi_1)$$
$$Y_x(\varphi_1,\varphi_2) = -Y_x(-\varphi_1,-\varphi_2)\cdot$$
$$Y_z(\varphi_1,\varphi_2) = Y_z(-\varphi_1,-\varphi_1)$$

Consequently, we need to solve the following inequality system:

$\begin{cases}(\sin\phi_2 - \sin\phi_1) \geq 0\\ (\cos\phi_2 - \cos\phi_1) \geq 0\end{cases}$, applying the famous trigonometric transformation:

$$\begin{cases}\sin\dfrac{\varphi_2-\varphi_1}{2}\cos\dfrac{\varphi_2+\varphi_1}{2} \geq 0\\ \sin\dfrac{\varphi_2+\varphi_1}{2}\sin\dfrac{\varphi_2-\varphi_1}{2} \geq 0\end{cases}$$, which is the equivalent of:

$$\left[\begin{array}{l}\begin{cases}\sin\dfrac{\varphi_2-\varphi_1}{2} \geq 0\\ \cos\dfrac{\varphi_2+\varphi_1}{2} \geq 0\\ \sin\dfrac{\varphi_2+\varphi_1}{2} \leq 0\end{cases}\\ \begin{cases}\sin\dfrac{\varphi_2-\varphi_1}{2} < 0\\ \cos\dfrac{\varphi_2+\varphi_1}{2} < 0\\ \sin\dfrac{\varphi_2+\varphi_1}{2} > 0\end{cases}\end{array}\right. \Leftrightarrow \begin{cases}\varphi_1 \leq \varphi_2 + 2\pi k\\ \varphi_1 \geq \varphi_2 - 2\pi + 2\pi k\\ \varphi_1 \geq -\pi - \varphi_2 + 2\pi k\\ \varphi_1 \leq -\varphi_2 + 2\pi k\end{cases}$$, where $k \in Z$.

In our problem, we may limit ourselves by the main values only, so the required φ_1, φ_2 may be in the following rectangle:

$$\begin{cases} \varphi_1 \le \varphi_2 + 2\pi \\ \varphi_1 \ge \varphi_2 \\ \varphi_1 \ge -\varphi_2 + \pi \\ \varphi_1 \le -\varphi_2 + 2\pi \end{cases}$$

We have chosen k=1, as the angles are positive in that case. It remains to cover the rectangle with a network and to obtain the results table.

For future work we need to sort the obtained results out in relation to Y_X, Y_Z, i.e. each table line should be sorted according to Y_Z and according to Y_X inside the line. This is rather a time-consuming procedure, but the time factor is not very important at the simulation stage. The table made in such a manner allows applying the fast searching algorithm with dichotomy method when we search for the required grade of optical wedges; the dichotomy method functions in the sorted data arrays only.

The dichotomy method is as follows: at each algorithm step we choose the medial element of the data chain and compare it to the required one. If the required element is less, we choose the left part of the chain, if it is greater – the right one, moreover, the medial element is not included in the chosen half because there is no need in the repeated comparison. Next we repeat the algorithm but use the chosen half as the data chain. If the elements are equal at the comparison stage, the medial element is a required one and the algorithm is completed.

However, the algorithm should be modified a little to apply the dichotomy method in our problem solution. The point is that we have to find an element in the table, which is mostly approximated, but not necessarily equal to the input optical beam deviation angles. It is connected with the fact that our table has only discrete values, while continuous values will naturally be given to the input of our control algorithm. Besides, we do not know beforehand how our required element will differ from the closest one, in other words, we cannot make approximation before the search stage because the sample spacing is quite possibly will not be constant.

To apply the dichotomy algorithm for searching for the closest element, we need to include the medial element into the chosen half because at the comparison point we do not know if the element is the closest to the required one, we can only omit the elements that are evidently farther than the medial one. At the moment when the chain consists of two elements only, we compare both elements with the required one and find the closest one. As a result, it is possible that one element is twice compared to the required one, being a medial element at some cycle iteration for the first time and at the last stage for the second time. However, neither this circumstance nor the inclusion of the medial elements into the subchains can influence on the algorithm performance.

It should be noted, that the control systems of the optical-radar systems certainly belong to the real-time systems in which the time factor is critical from the point of disturbance recognition to the point of its stabilization. But the application of the simulation modelling algorithms in such-like systems allows shifting the computation to the modelling stage thus providing the essential time advantage.

Demin Anatoliy is the head of the Optical-digital Systems and Technologies Department (primary department of ITMO University affiliated with "LOMO" JSC), Doctor of Engineering, full professor in Saint Petersburg National Research University of Information Technologies, Mechanics and Optics. Corresponding member of Peters Academy of Arts and Sciences. Deputy Chair of D 212.227.05 Dissertation Council, a member of D 212.227.06 Dissertation Council.

e-mail: dav_60@mail.ru

Area of expertise: physical and applied optics, mathematical methods and means of analytical modelling and computer-based simulation, systems theory, control engineering, measurement theory.

Keywords: *algorithm, model, dynamic scene of simulation modelling, remote sensing, optical range, radiation spectrum, atmosphere physical properties, trajectory, hierarchical multi-parameter dynamic engineering system, forecasting.*

Has more than 150 publications including 3 monographs and 31 invention certificates and patents. More than 30 research scientific works and R&D projects were executed under the academic advising of A.V. Demin in ITMO University in 1978-2014.

The algorithms for computer simulation of optical information-measuring and controlling arrays developed by him allowed reducing the material-technical and financial costs at making the design decisions, and the engineering systems based on the design decisions are successfully implemented in various-purpose optical and optoelectron devices for research scientific works and R&D projects.

Won some Russian Federation government and industry awards.

Dmitrieva Svetlana - postgraduate student of the Department of Optical-digital Systems and Technologies (primary department of ITMO University affiliated with “LOMO” JSC). Scientific supervisor - Doctor of Engineering, full professor Demin A.V.

E-mail: job_**8**@mail.ru

Area of expertise: mathematical methods and analytical tools and computer modeling.

Keywords: *algorithm, model, remote sensing, optical wavelengths, emission spectrum, the physical parameters of the atmosphere, orbit, hierarchical multivariable dynamic technical system prediction.*

Has more than 30 publications, including 2 manuals.

Demin Anatoliy

Dmitrieva Svetlana

APPLYING MATH MODELLING METHODS FOR FORECASTING THE ENGINEERING SYSTEM STATES

Educational book

В авторской редакции
Редакционно-издательский отдел Университета ИТМО
Зав. РИО Н.Ф. Гусарова
Подписано к печати
Заказ №
Тираж
Отпечатано на ризографе