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,  
  
• •  
  
**MathCAD**



681.7:53.081.5

MathCAD  
: , 2006. 101 .

MathCAD.

« » « - ».

© - , 2006

© . . , 2006

“

200200 –

200203 – -

1.	.....	4
1.	.....	6
2.	MathCAD .....	8
2.1.	.....	8
2.2.	.....	9
2.3.	.....	10
2.4.	.....	11
2.5.	- .....	13
2.6.	MathCAD .....	14
3.	- .....	18
4.	.....	19
5.	.....	20
5.1.	.....	22
5.2.	.....	23
5.3.	.....	24
6.	.....	30
6.1.	.....	30
6.2.	.....	32
6.3.	MathCAD.....	35
7.	.....	39
8.	( ..... ).....	46
8.1.	Ving.....	55
8.2.	( ..... ).....	58
9.	.....	61
9.1.	.....	66
9.2.	( ..... ).....	75
10.	.....	77
10.1.	.....	77
10.2.	.....	78
	1.....	89
	2.....	93
	3.....	94
	4.....	95
	5.....	98
	.....	99

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... [1,2].  
[2-3].

MathCAD.

MathCAD.

**1.**

IX  
” – “Algoritmi de numero Indorum”.

- 7 :
- 1)
  - 2)
  - 3) —
  - 4) —
  - 5) —
  - 6) —
  - 7) —
- (  
..).
- ( )

- 1) ( );
- 2) ;
- 3) ;
- 4) - .

Fortran, Algol, -1, Basic, Pascal .  
IBM

FORTTRAN,

LISP.

Progol.

(Java, C<sup>++</sup>).

2.

**MathCAD**

MathCAD

C<sup>++</sup>

MathCAD –

MathCAD

MathCAD

++  
MathCAD

( ).

MathCAD.

( , , ) ,

*.mcd.*

### 2.1.

$10^{-307}$   $10^{307}$ .

$-o,$

$-h.$

*b*

$$i := \sqrt{-1}.$$

: "*<string>*".



$$acre = 4046.856 \text{ m}^2, \quad atm = 101325 \text{ Pa}, \quad g = 9.807 \cdot \frac{m}{s^2}.$$

## 2.2.

— ( ),  
 ,  
 .  
 :  
 $a := A$      $a \leftarrow A$   
 ,  
 $a -$  , —  
 “:=”  
 , “←”  
 .  
 , ( ),  
 ( ).  
 : ,  
 , , , , ,  
 , , , :  
 $a := 5, a := 5., a := 5 + 2i, a := \frac{2}{5}, a := \frac{2}{5} \frac{1}{2}, a := "d", a := 2 \neq 1.$

“ ” “ ”

( ),

## 2.3.

MathCAD—

`rnorm(N,M, ) runif(N,a,b).`  
`N-`

`a:=b,c..d.,`  
`b,`  
`< d.`

`i:=0..6 a[i]:=i^2 a^T=(0 1 4 9 16 25 36)`

1) `a:=if(`  
`if`  
`= 1,`  
`1, - 2.`

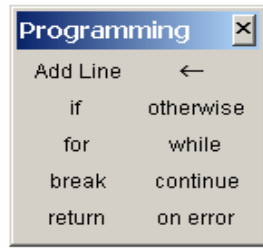
2) `a:=`  
`1 on error`  
`2.`  
`1,`  
`2.`

`f(x) :=  $\frac{\sin(x)}{x}$  on error 1 f(0) = 1`

**2.4.**

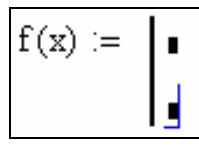
`.1. "Math" ( )`

:



.2.

: "Add Line" ( ),



.3.

*f*

*x*.

"Add Line".

"←".

- "{".

"if"

"otherwise"  
"if"

"otherwise"  
"if"

"continue".

"for" -

"while"

, "while" -

"break"

"return"

. "break" -

"return" -

"on error".

error(" ").

(.3).

a := 2	a = 2	
b <sub>1</sub> := 1	b = 0 1	a := 8
c := a-b	c = 0 8	d := $\left  \begin{array}{l} a \leftarrow 4 \\ c \\ a \end{array} \right.$
d = 0 2	a = 8	

.4.

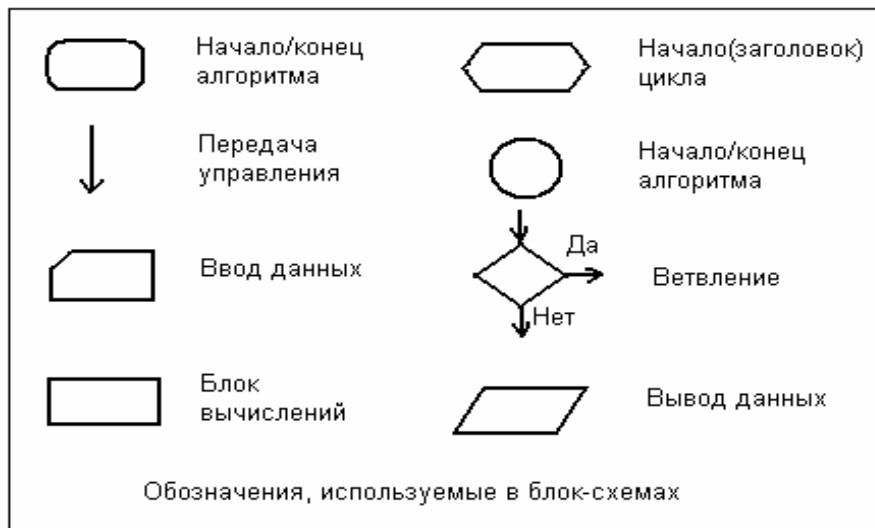
*MathCAD,*

2.

8.

-8.

2.5.



.5.

2.6.

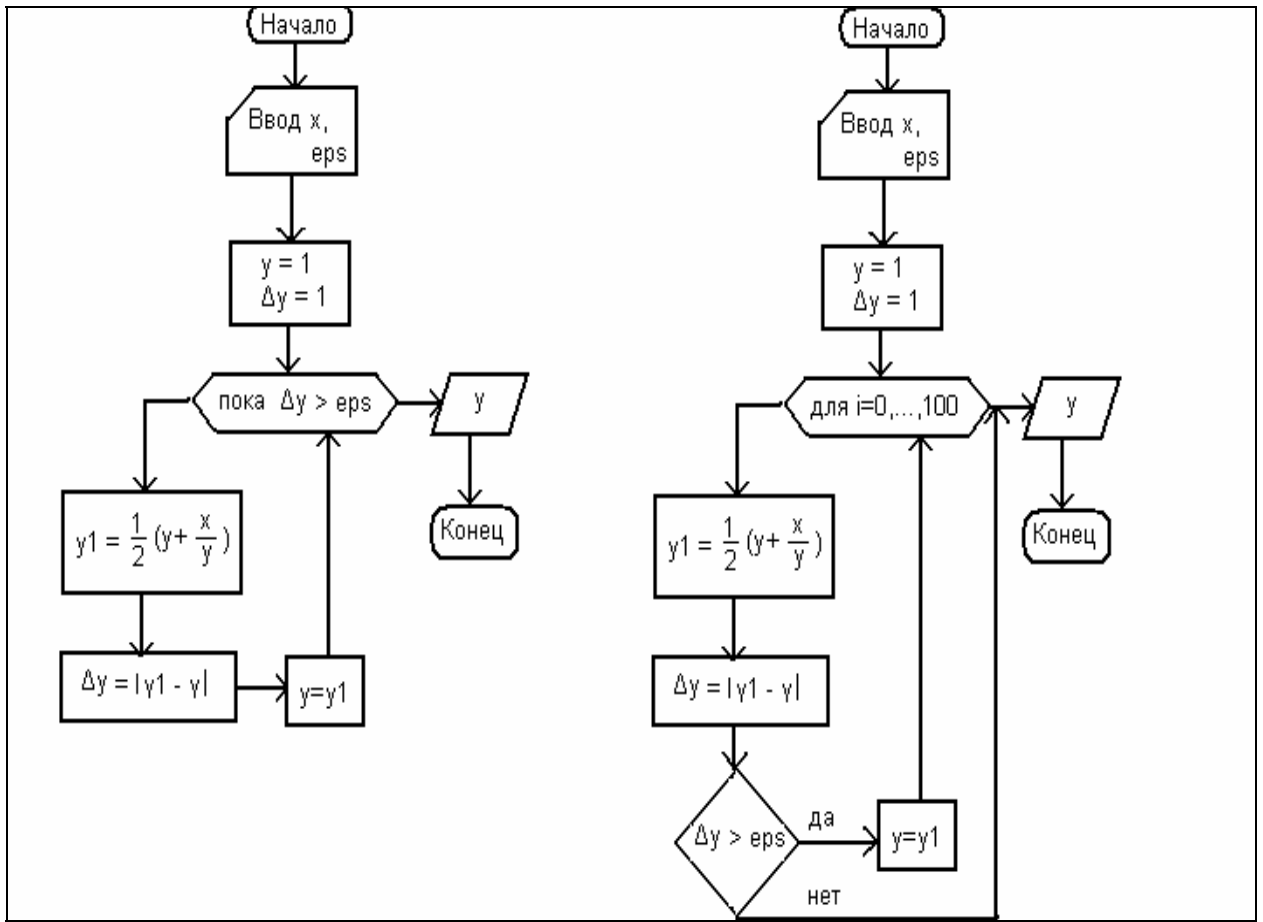
**MathCAD**

1.

$x$

$$y_{n+1} = \frac{1}{2} y_n + \frac{x}{y_n}, \quad y_0 = 1.$$

$y < eps - eps -$



.6.

“ ”,

*while:*

<p>RootWhile(x, eps) :=</p> <p>Квадрат числа</p> <p>Погрешность вычислений</p>	<p><math>y \leftarrow 1</math> Нулевое приближение</p> <p><math>\Delta y \leftarrow 1</math> Начальное значение разности последовательных вычислений</p> <p>while <math>\Delta y &gt; eps</math> Заголовок цикла</p> <p style="margin-left: 20px;"><math>t \leftarrow \frac{1}{2} \left( y + \frac{x}{y} \right)</math> Текущее значение, вычисленное по формуле Герона</p> <p style="margin-left: 20px;"><math>\Delta y \leftarrow  y - t </math> Текущее значение разности вычислений</p> <p style="margin-left: 20px;"><math>y \leftarrow t</math> Текущее значение корня</p> <p>У Вывод вычисленного значения</p>
--	---

.7.

*while:*

```

RootFor(x, eps) :=
  y ← 1
  Δy ← 1
  for i ∈ 0..106
    Заголовок цикла for
    t ←  $\frac{1}{2} \cdot \left( y + \frac{x}{y} \right)$ 
    Δy ← |y - t|
    y ← t
    if Δy < eps
      инструкция if с составным
      вычислительным блоком
      k ← i
      break
      число итераций
  (k, y)
  вывод информации
  в виде вектора

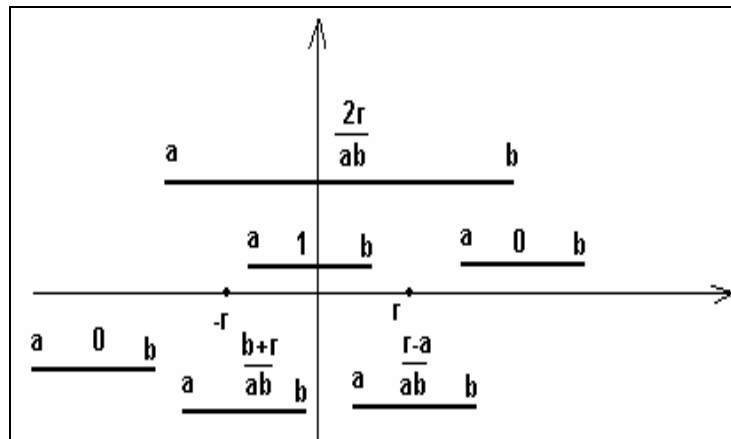
```

.8.

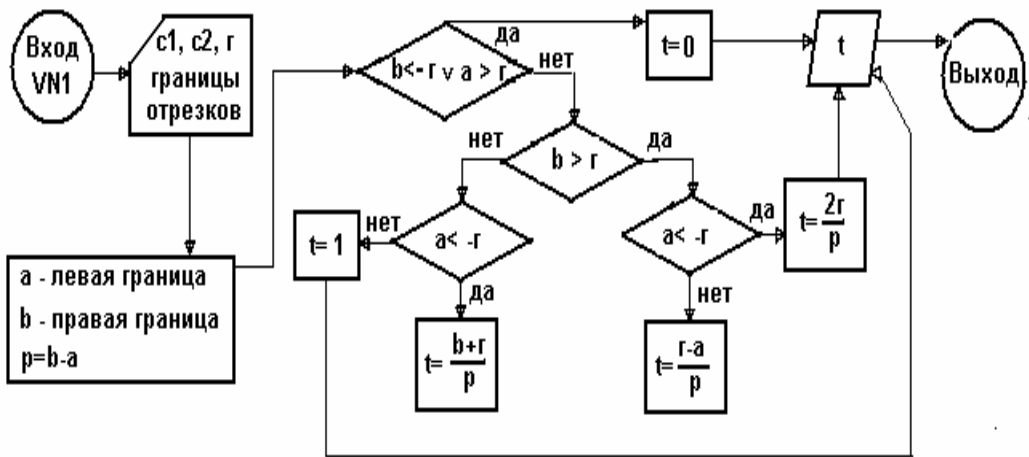
for:

2.  $[a, b]$   $[b, a]$   $[-r, r]$   $a$   $b$

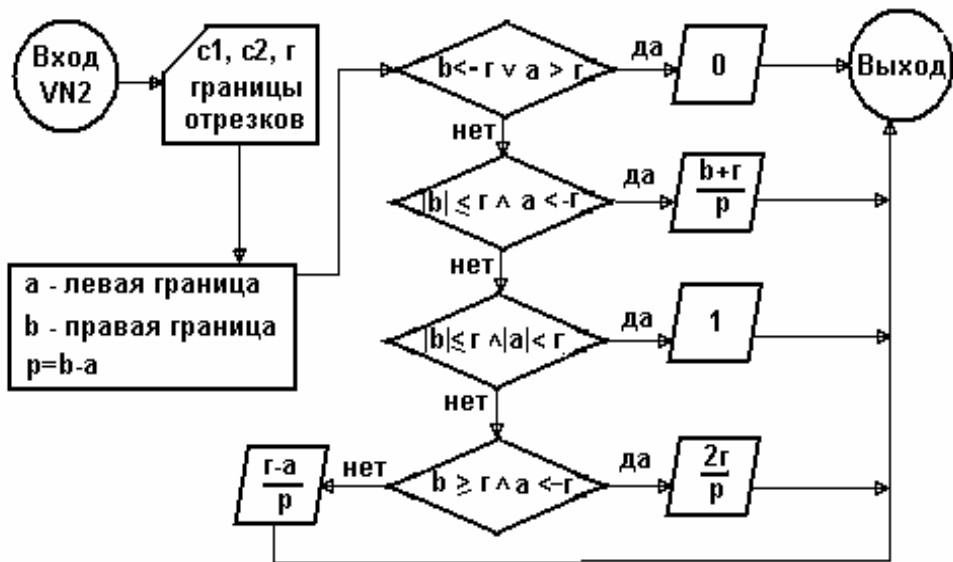
.9.



.9.



.10. - 2, 1



.11. - 2, 2



$\text{Vn1}(c1,c2,r) := \begin{cases} a \leftarrow \min(c1,c2) \\ b \leftarrow \max(c1,c2) \\ p \leftarrow b - a \\ \text{return } 0 \text{ if } b < -r \vee a > r \\ \text{return } \frac{b+r}{p} \text{ if }  b  \leq r \wedge a < -r \\ \text{return } 1 \text{ if }  b  \leq r \wedge  a  \leq r \\ \text{return } \frac{2r}{p} \text{ if } b > r \wedge a < -r \\ \frac{r-a}{p} \end{cases}$	$\text{Vn2}(a,b,r) := \begin{cases} c \leftarrow \min(a,b) \\ d \leftarrow \max(a,b) \\ p \leftarrow d - c \\ t \leftarrow 0 \text{ if } d < -r \vee c > r \\ \text{otherwise} \\ \begin{cases} \text{if } d \geq r \\ \begin{cases} p \leftarrow \frac{2r}{p} \text{ if } c < -r \\ t \leftarrow \frac{r-c}{p} \text{ otherwise} \end{cases} \\ \text{otherwise} \\ \begin{cases} t \leftarrow \frac{d+r}{p} \text{ if } c < -r \\ t \leftarrow 1 \text{ otherwise} \end{cases} \end{cases} \\ t \end{cases}$
---	--

.12

( ( ) ),

```

Dv(c,a,b) :=
  delta ← |a-b| / 1000
  x ← delta
  while Vn1(a,b,x) < c
    x ← x + delta
  x

```

.13.

( ), “ : ” (a,b).

MathCAD :

) ;

) ;

) ;

) ;

) ;

3.

## MathCAD

$N$ ,  $M$   
 $($  -  $)$   
 $\text{rnorm.}$

$$N := 3 \quad M := 1 \quad \sigma := 0.5 \quad V := \text{rnorm}(N, M, \sigma) \quad V = \begin{pmatrix} 0.781 \\ 0.66 \\ 0.763 \end{pmatrix}$$

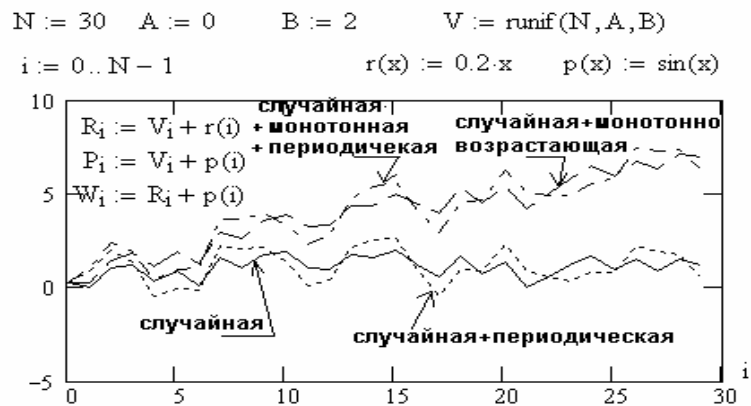
$\text{runif.}$   
 $N$ ,  
 $< B$ .

$$N := 3 \quad A := 0 \quad B := 2 \quad V := \text{runif}(N, A, B) \quad V = \begin{pmatrix} 0.183 \\ 0.295 \\ 1.977 \end{pmatrix}$$

## V MathCAD

$\text{mean}(V)$  -  
 $\text{stdev}(V)$  -  
 $\text{skew}(V)$  -  
 $\text{min}(V)$  -  
 $\text{max}(V)$  -  
 $\text{sort}(V)$  -  
 $\text{csort}(V, 0)$  -

$3\sigma$



.14.

4.

[2].

, 1000 , 2786-76 0,3 – 0,01%,

[2]

$$\frac{\Delta r}{r} = 0,001 - 0,01.$$

$N$

$D$ .

$$R \approx 450 \frac{D^2}{N}.$$

0,01 1,0 .

3514-76

$n=0,0002-0,002.$

5.

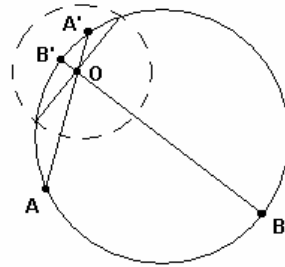
?

“ ”

:

“ ”

( .15).



.15.

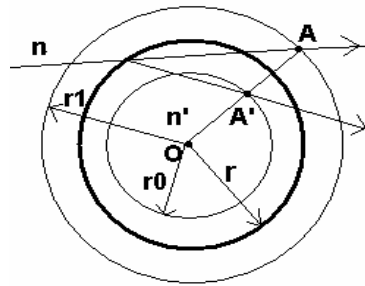
“ ”

$n_0$

$$n(r) = \frac{n_0}{1 + \frac{r}{a}}$$

(1)

.16.



.16.

$r_0$   $r_1$  ,

$$r = r_0 r_1, \quad r_0 = \frac{n}{n'}, \quad r_1 = \frac{n'}{n}. \quad (2)$$

$$x' = \frac{F_1(x, y, z)}{F_0(x, y, z)}, \quad y' = \frac{F_2(x, y, z)}{F_0(x, y, z)}, \quad z' = \frac{F_3(x, y, z)}{F_0(x, y, z)}, \quad (3)$$

$$F_i = a_i x + b_i y + c_i z + d_i$$

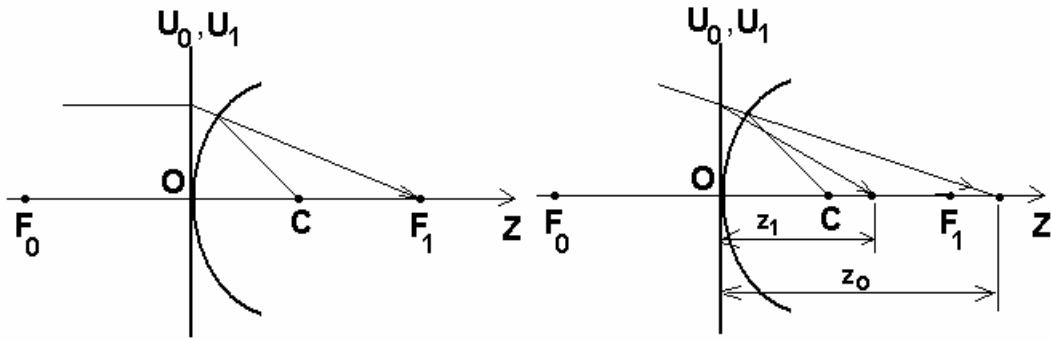
15.

$$y_1 = \frac{f}{z_0} y_0, \quad x_1 = \frac{f}{z_0} x_0, \quad z_1 = \frac{f \cdot f'}{z_0}. \quad (4)$$

**5.1.**

OZ.

(.17):



.17.

( )  
( )

$$n_0 \frac{1}{r} - \frac{1}{z_0} = n_1 \frac{1}{r} - \frac{1}{z_1} \quad (5)$$

$$n_0 \quad n_1 -$$

$$r -$$

$$, z_0 - Z-$$

$$, z_1 -$$

( )

$$\frac{n_1}{z_1} - \frac{n_0}{z_0} = \frac{n_1 - n_0}{r} = \Phi \quad (6)$$

$$z_0 \quad z_1$$

$$F_0 = -\frac{n_0 r}{n_1 - n_0}, \quad F_1 = \frac{n_1 r}{n_1 - n_0}. \quad (7)$$

7427-76,

$$f_0 = -\frac{n_0 r}{n_1 - n_0}, \quad f_1 = \frac{n_1 r}{n_1 - n_0}. \quad (8)$$

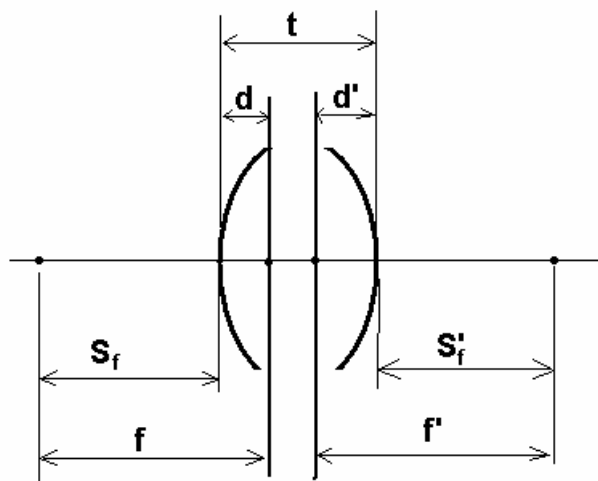
$$-\frac{n_0}{f_0} = \frac{n_1}{f_1} = \Phi. \quad (9)$$

## 5.2.

(.21).

$$\Phi = \Phi_1 + \Phi_2 - \frac{t}{n_1} \Phi_1 \Phi_2, \quad (10)$$

$n_1 -$



.18.

$$f' = -\frac{n \cdot r_1 \cdot r_2}{(n-1) \cdot [n \cdot (r_1 - r_2) - (n-1) \cdot t]} \quad (11)$$

$$d = \frac{n-1}{n \cdot r_2} t \cdot f', \quad d' = \frac{n-1}{n \cdot r_1} t \cdot f'$$

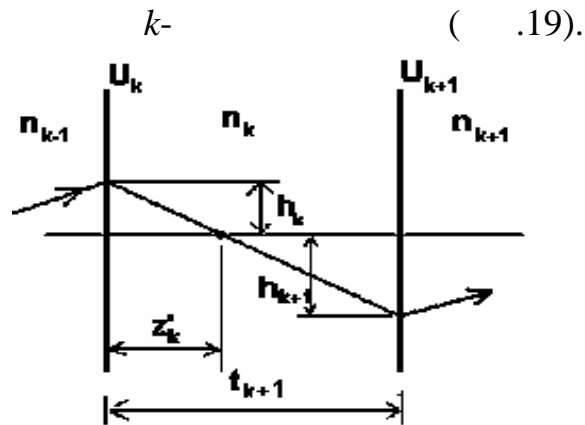
5.3.

$h_k,$

(.17).  $h_{k+1}$  :

$$h_{k+1} = h_k \left( 1 - \frac{t_k}{z'_k} \right), \quad (12)$$

$z'_k -$



.19.

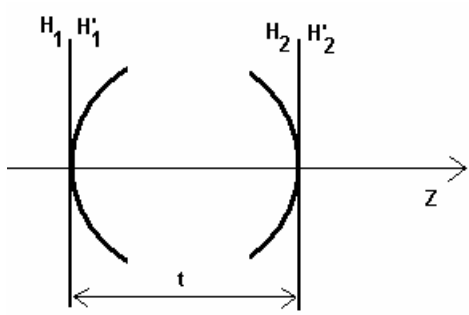
(10)

$t$



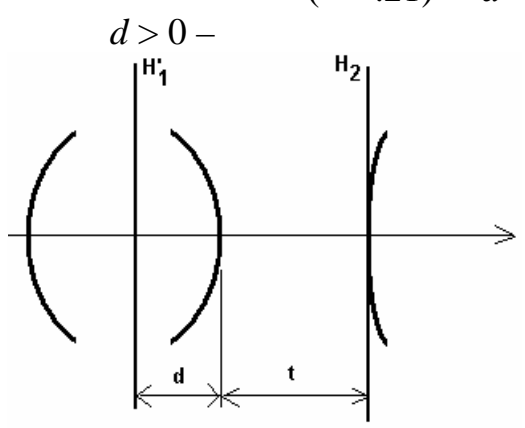
(10).

(.20),  $t -$



.20.  
(10)

(.21)  $d < 0,$



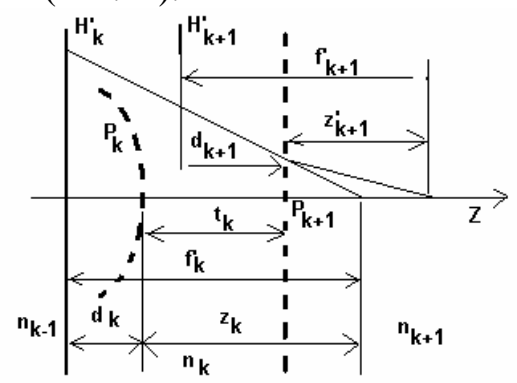
$d,$   
 $d < 0.$

.21.

$d = 0$

$z'$   $d$

(.22).



.22.

$$1) \quad z_k = \frac{h_k}{\alpha_{k+1}}, \quad (13)$$

$$2) \quad f_k = \frac{n_{k+1}}{\Phi}, \quad (14)$$

$$3) \quad d_{k+1} = z_k - f_k, \quad (15)$$

$$4) \quad t \equiv t_k - d_k, \quad (16)$$

$$\alpha_k = \frac{h_k}{z_k}. \quad (17)$$

(6), (10), (12), (13)-(16)

(6) (12)

(17).

$$1) \quad \Phi_k = \frac{n_k - n_{k-1}}{r_k} \quad 2) \quad \alpha_{k+1} = \frac{1}{n_{k+1}} (n_k \alpha_k + h_k \Phi_k) \quad 3) \quad h_{k+1} = h_k - \alpha_{k+1} t_{k+1}$$

$$4) \quad z_k = \frac{h_k}{\alpha_{k+1}} \quad 5) \quad \Phi = \Phi + \Phi_k - \frac{t_k - d_k}{n_k} \Phi \Phi_k$$

$$6) \quad f_k = \frac{n_{k+1}}{\Phi} \quad 7) \quad d_{k+1} = f_k - z_k \quad (18)$$

Syst:

$$\text{Syst} := \begin{pmatrix} R_0 & R_1 & \dots & R_{N-1} & D \\ t_0 & t_1 & \dots & t_{N-1} & Z_0 \\ n_0 & n_1 & \dots & n_{N-1} & n_N \end{pmatrix} \quad (19)$$

$N+1$  ,  $N -$

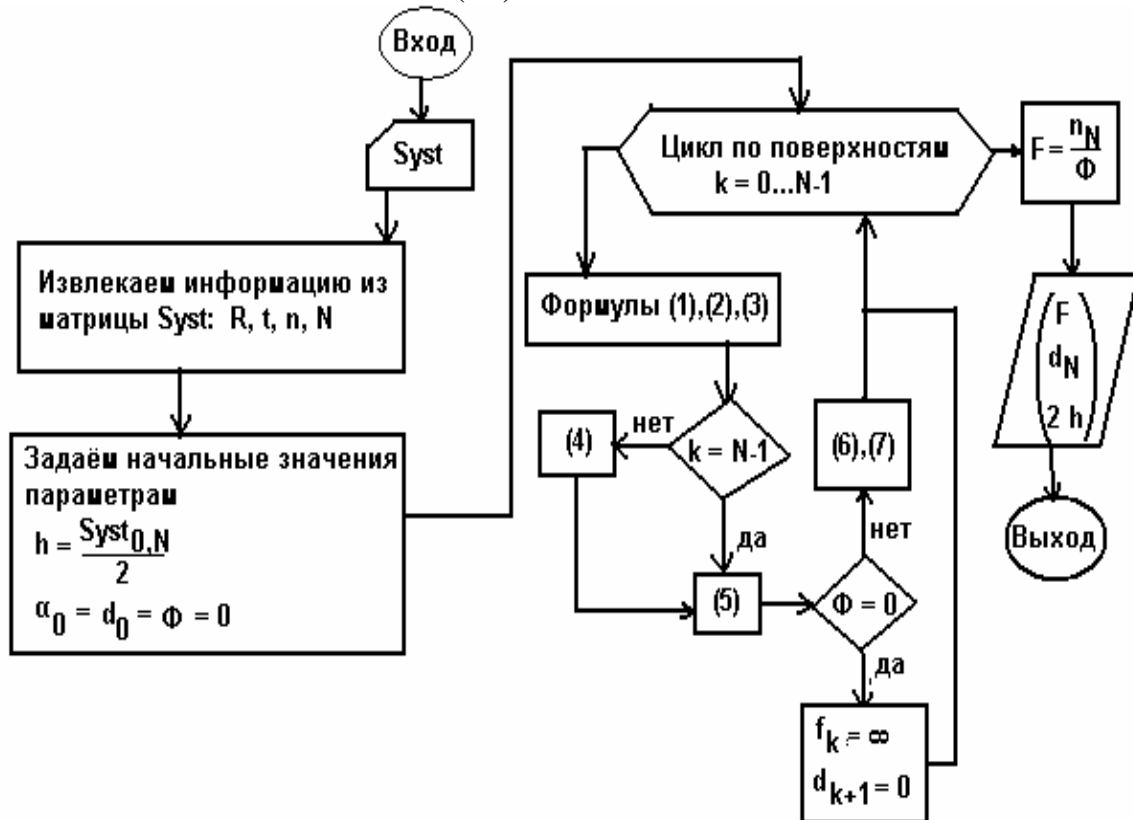
$(t_0=0)$

$D,$   $Z_0$

$t_0=0, Z_0=\infty.$

Gauss(Syst)

(18).



.23.

ardinal

(11).

100

10

MathCAD

Проверка

$$f(r_1, r_2, n, d) := \frac{-n \cdot r_1 \cdot r_2}{(n-1) \cdot [n \cdot (r_1 - r_2) - (n-1) \cdot d]}$$

$$f(100, -100, 1.5, 10) = 101.695$$

$$d'(r_1, r_2, n, d) := \frac{n-1}{n \cdot r_1} \cdot d \cdot f(r_1, r_2, n, d)$$

$$d'(100, -100, 1.5, 10) = 3.39$$

$$\text{Cardinal}(S) = \begin{pmatrix} 101.695 \\ 3.39 \\ (2,1) \end{pmatrix}$$

$$\begin{array}{l}
\text{Gauss}(\text{Syst}) := \left\{ \begin{array}{l}
r \leftarrow ((\text{Syst})^T)^{\langle 0 \rangle} \\
t \leftarrow ((\text{Syst})^T)^{\langle 1 \rangle} \\
n \leftarrow ((\text{Syst})^T)^{\langle 2 \rangle} \\
N \leftarrow \text{cols}(\text{Syst}) - 1 \\
h_0 \leftarrow \frac{\text{Syst}_{0, N}}{2} \\
\alpha_0 \leftarrow 0 \\
\Phi \leftarrow 0 \\
d_0 \leftarrow 0 \\
\text{for } k \in 0..N-1 \\
\left\{ \begin{array}{l}
\phi_k \leftarrow \text{if} \left( r_k = \infty, 0, \frac{n_{k+1} - n_k}{r_k} \right) \\
\alpha_{k+1} \leftarrow \frac{1}{n_{k+1}} \cdot (n_k \cdot \alpha_k + h_k \cdot \phi_k) \\
z_k \leftarrow \text{if} \left( \alpha_{k+1} \neq 0, \frac{h_k}{\alpha_{k+1}}, \infty \right) \\
h_{k+1} \leftarrow h_k - \alpha_{k+1} \cdot t_{k+1} \text{ if } k \neq N-1 \\
\Phi \leftarrow \Phi + \phi_k - \frac{t_k - d_k}{n_k} \cdot \Phi \cdot \phi_k \\
f_k \leftarrow \text{if} \left( \Phi \neq 0, \frac{n_{k+1}}{\Phi}, \infty \right) \\
d_{k+1} \leftarrow \text{if} \left( \Phi \neq 0, z_k - f_k, 0 \right)
\end{array} \right. \\
F \leftarrow \frac{n_N}{\Phi} \\
\left( \begin{array}{c}
F \\
z_{N-1} \\
2 \cdot h
\end{array} \right)
\end{array} \right.
\end{array}$$

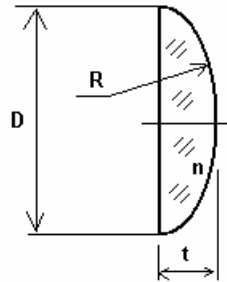
.24.

*Gauss*

6.

## 6.1.

[1, c .77].



$$R = -500, t = 10, D = 30, n = 1,5 \quad (11)$$

$$f' = \frac{R}{n-1}, \quad (20)$$

$$A_R = \frac{-1}{n-1}, \quad (21)$$

$$\text{Syst} := \begin{pmatrix} \infty & R & D \\ 0 & t & \infty \\ 1 & n & 1 \end{pmatrix}$$

$$\text{Syst1} := \begin{pmatrix} \infty & R + \Delta & D \\ 0 & t & \infty \\ 1 & n & 1 \end{pmatrix}$$

Gauss

$$A(\Delta) := \frac{\text{Gauss}(\text{Syst1})_0 - \text{Gauss}(\text{Syst})_0}{\Delta}$$

Gauss

(21)

$$A(\ ) = -2.$$

$$\Delta = \frac{D^2}{8R_0}, \tag{22}$$

$$R_0 \approx 450 \frac{D^2}{N}.$$

$$1 \quad 10,$$

$$D = 30$$

$$: |R_0| = 4 \cdot 10^4 - 4 \cdot 10^5$$

$$\text{Syst1} := \begin{pmatrix} R_0 & r & D \\ 0 & t & \infty \\ 1 & n & 1 \end{pmatrix}.$$

ardinal

$$A(R_{02}) := \frac{\text{Gauss}(\text{Syst0})_0 - \text{Gauss}(\text{Syst})_0}{D^2} 8R_{02} \quad A(R_{02}) = 8.647 \times 10^3$$

$$R_{02} = 4 \times 10^5 \quad D = 30$$

## 6.2.

[2, . 205]:

1)

2)

0,7),

$$\rho = \pm 1.$$

$$= 0.$$

(| | >

3)

4)

5)

$$\theta = \min_{i=1, \dots, m} \left( \sqrt{\sum_{i=1}^m \theta_i^2} \right) \quad (23)$$

( ), k P. 1.

P	k
0,90	0,95
0,95	1,1
0,99	1,4

6)

$$s^2 = \frac{1}{N-1} \sum_{k=1}^N \chi_k^2 = \frac{1}{N-1} \sum_{k=1}^N (y_k - \bar{y})^2, \quad \bar{y} = \frac{1}{N} \sum_{k=1}^N y_k, \quad y(x) \quad (24)$$

$$s_{\Sigma}^2 = \sum_{i=1}^m s_i^2 + 2 \sum_{i < j} \rho_{i,j} s_i s_j \quad (25)$$

$$s^2 = s_1^2 + s_2^2 + 2\rho \cdot s_1 s_2 \quad (25)$$

$$\rho = \frac{s^2 - s_1^2 - s_2^2}{2s_1 s_2} \quad (26)$$

(26).

7)

8.207-76

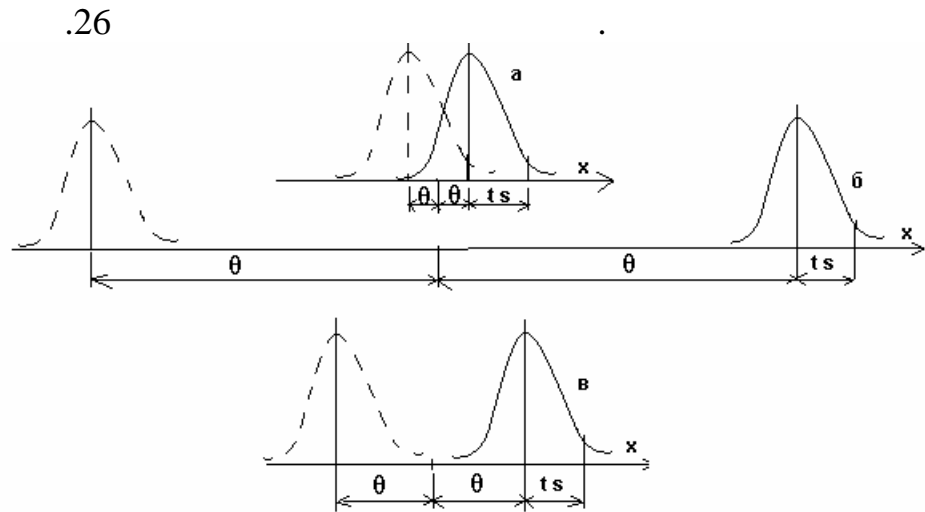
a)

$$< 0,8 s, \quad (27)$$



$$\begin{aligned}
 & ) \\
 & > 8 s, \tag{28} \\
 & , \quad , \quad \cdot \\
 & )
 \end{aligned}$$

$$\Delta = 2(|\theta| + t \cdot s), \tag{29}$$



.26.

$M=200.$

$(-R_{01}, -R_{02}) Y (R_{01}, R_{02}),$   
 $R_{01}, R_{02} -$   
 $N=1 \quad 10. \quad , \quad R_0 \approx 450 \frac{D^2}{N}.$   
 MathCAD,

$M := 200$	$R_{01} := 4 \cdot 10^4$	$R_{02} := 4 \cdot 10^5$					
$r1 := \text{runif}\left(\frac{M}{2}, -R_{02}, -R_{01}\right)$	$r2 := \text{runif}\left(\frac{M}{2}, R_{01}, R_{02}\right)$	$r0 := \text{stack}(r1, r2)$					
$r0^T =$	0	1	2	3	4	5	
	0	$-1.067 \cdot 10^5$	$-3.931 \cdot 10^5$	$-2.854 \cdot 10^5$	$-2.053 \cdot 10^5$	$-8.658 \cdot 10^4$	$-1.319 \cdot 10^5$

runif,

$M$

stack.

$R.$   
 $\in (0,001; 0,01)R.$  , 90%

$$s = \frac{\Delta}{1,6}. \tag{30}$$

MathCAD,

$R := -100$	$\Delta := 0.01 \cdot  R $	$\sigma := \frac{\Delta}{1.6}$	$r1 := \text{rnorm}(M, R, \sigma)$					
$r1^T =$	0	1	2	3	4	5	6	
	0	-100.091	-100.586	-100.08	-99.635	-99.165	-99.954	-101.264

rnorm,

$i := 0..M-1$     $n := 1.5$     $t := 10$     $D := 20$

$$\text{Syst} := \begin{pmatrix} \infty & R & D \\ 0 & t & \infty \\ 1 & n & 1 \end{pmatrix} \quad \text{Syst0}_i := \begin{pmatrix} r0_i & R & D \\ 0 & t & \infty \\ 1 & n & 1 \end{pmatrix} \quad \text{Syst1}_i := \begin{pmatrix} \infty & r1_i & D \\ 0 & t & \infty \\ 1 & n & 1 \end{pmatrix} \quad \text{Syst2}_i := \begin{pmatrix} r0_i & r1_i & D \\ 0 & t & \infty \\ 1 & n & 1 \end{pmatrix}$$

$i.$

ardinal

stdev.

$$\begin{aligned}
F &:= \text{Cardinal}(\text{Syst})_0 & F &= 1 \times 10^3 \\
\delta 0_i &:= \text{Cardinal}(\text{Syst}0_i)_0 - F & \sigma 0 &:= \text{stdev}(\delta 0) & \sigma 0 &= 3.965 \\
\delta 1_i &:= \text{Cardinal}(\text{Syst}1_i)_0 - F & \sigma 1 &:= \text{stdev}(\delta 1) & \sigma 1 &= 5.965 \\
\delta 2_i &:= \text{Cardinal}(\text{Syst}2_i)_0 - F & \sigma 2 &:= \text{stdev}(\delta 2) & \sigma 2 &= 7.329 \\
\rho &:= \frac{\sigma 2^2 - \sigma 1^2 - \sigma 0^2}{2\sigma 0 \cdot \sigma 1} & \rho &= 0.051
\end{aligned}$$

### 6.3.

### MathCAD

$$Y = \{Y_1, Y_2, \dots, Y_n\}.$$

1.

$$Y_s = \text{scort}(Y, 0).$$

2.

$Y$ :

$$Y_{\min} = \min(Y), \quad Y_{\max} = \max(Y), \quad RY = Y_{\max} - Y_{\min}.$$

3.

$m$ .

$$m_{\min} = 0,55n^{0,4}, \quad m_{\max} = 25/11m_{\min},$$

floor

$$m = \text{floor} \frac{m_{\min} + m_{\max}}{2},$$

if,

$$m = \text{if} \left( m - 2 \text{floor} \frac{m}{2} = 0, m + 1, m \right).$$

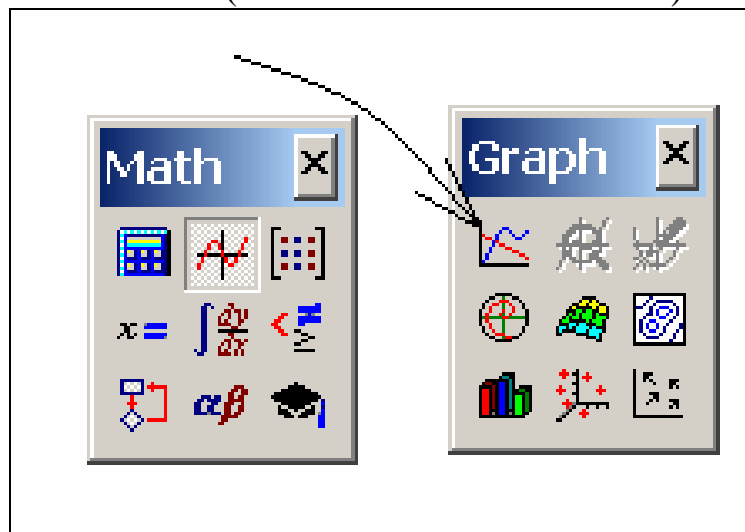
4.  $I = \frac{RY + 10^{-9}}{m}$ .

5.  $j=0..m$  :  $int_j = Y_{min} + j*I$

6. hist,  $H = hist(int, Ys)$ .  $m-1$  int

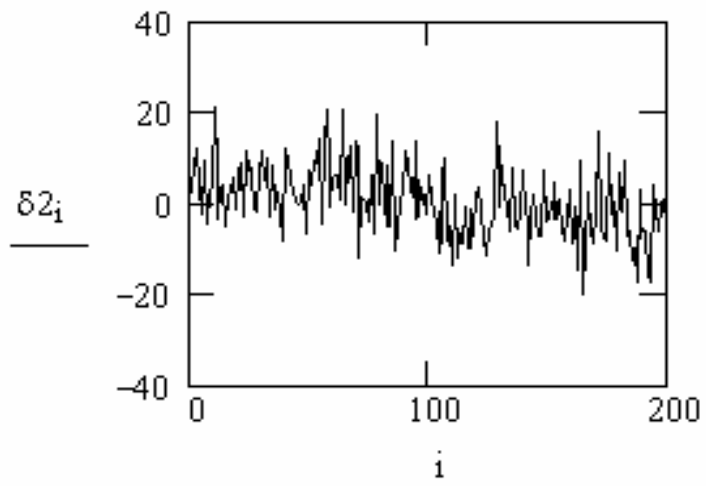
7.  $H_j(int_j + I/2)$ ,  $H_j(j)$

MathCAD



.27. "Math" ( ) "Graph".

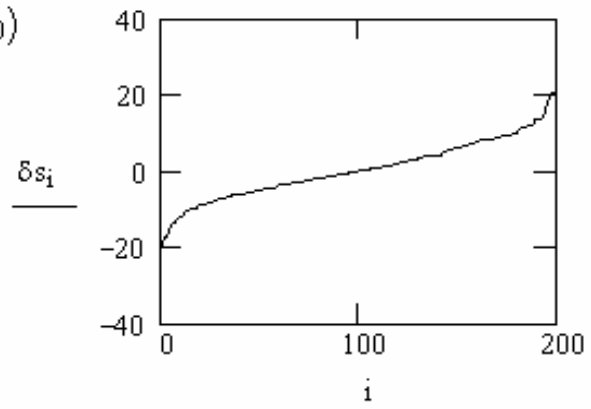
: @ (Shift+2)



.28.

, , : , - , , .

1)  $\delta s := \text{csort}(\delta 2, 0)$



.29.

:

$$2) \delta_{\min} := \min(\delta_2) \quad \delta_{\max} := \max(\delta_2) \quad R\delta := \delta_{\max} - \delta_{\min}$$

$$\delta_{\min} = -20.01 \quad \delta_{\max} = 21.101 \quad R\delta = 41.111$$

$$3) m_{\min} := 0.55 \cdot M^{0.4} \quad m_{\max} := \frac{25}{11} \cdot m_{\min} \quad m := \frac{\text{floor}(m_{\min} + m_{\max})}{2}$$

$$m_{\min} = 4.579 \quad m_{\max} = 10.407 \quad m = 7$$

$$4) I := \frac{R\delta + 10^{-9}}{m} \quad I = 5.873 \quad (I - \text{интервал или квант})$$

$$5) j := 0..m \quad \text{int}_j := \delta_{\min} + j \cdot I$$

$$6) H\delta := \text{hist}(\text{int}, \delta_s) \quad H\delta - \text{вектор частотностей попадания случайной величины}$$

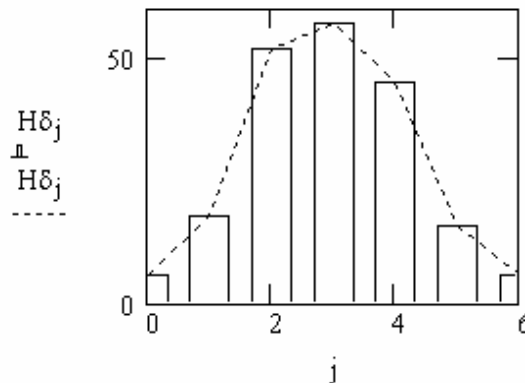
в построенные интервалы

$$H\delta^T = (6 \ 18 \ 52 \ 57 \ 45 \ 16 \ 6)$$

$$7) \delta_j := \text{int}_j + \frac{I}{2} \quad \delta_j - \text{середины интервалов}$$

“Graph” ( ), “Format” ( ), “Traces” ( ), “Type” ( ), “Bar” ( ). “OK”

*H* .



.30.

( $j=0,1,\dots,7$ ),  
MahtCAD

$H_m$

.25.

7.

(.31).

1)

2)

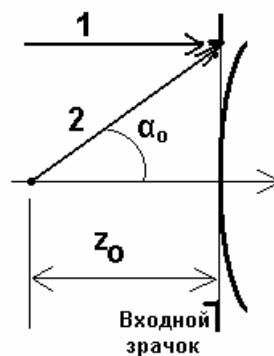
$$H = \frac{\pi \cdot \tau \cdot \sin^2(\alpha_0)}{K}, \quad K = V^2 -$$

$$K = V -$$

$$K = 1 -$$

(30)

1



.31.

$$\begin{aligned}
 & \text{Syst} = \begin{matrix} R_0 & R_1 & \dots & \infty & R_{k+1} & \dots & R_{N-1} & A & \beta \\ t_0 & t_1 & t_{k-1} & t_k & \dots & t_{N-1} & z_0 & z & \\ n_0 & n_1 & 1 & 1 & \dots & n_{N-1} & n_N & k & \end{matrix} \quad (31) \\
 & \qquad \qquad \qquad N \qquad \qquad \qquad ,
 \end{aligned}$$

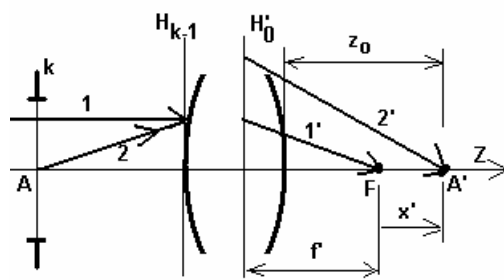
$$\begin{aligned}
 & k- \qquad \qquad \qquad R_k = \infty, \\
 & \qquad \qquad \qquad n=1. \qquad \qquad \qquad A \\
 & \qquad \qquad \qquad N-
 \end{aligned}$$

$$\begin{aligned}
 & \text{Cardinal,} \qquad \qquad \qquad : \\
 & \qquad \qquad \qquad = 0. \qquad \qquad \qquad (19)
 \end{aligned}$$

$$z_A = \text{Syst}_{1,N}, \quad \alpha_A = \frac{D}{2z_0}, \quad (32)$$

$$\begin{aligned}
 & N - \qquad \qquad \qquad : \\
 & \qquad \qquad \qquad ( \quad .32).
 \end{aligned}$$





.32.

$$A = A \cdot |V| = A \cdot \left| \frac{x'}{f'} \right| = A \left| \frac{z_k - z_f}{f'} \right|, \quad (33)$$

$V =$

.33.

Syst.

$$\begin{array}{l}
\text{Pupil}(\text{Sy}) := \left\{ \begin{array}{l}
r \leftarrow ((\text{Sy})^T)^{\langle 0 \rangle} \\
t \leftarrow ((\text{Sy})^T)^{\langle 1 \rangle} \\
n \leftarrow ((\text{Sy})^T)^{\langle 2 \rangle} \\
\delta\alpha \leftarrow (\text{Sy}^T)^{\langle 3 \rangle} \\
N \leftarrow \text{cols}(\text{Sy}) - 1 \\
\Phi \leftarrow 0 \\
d_0 \leftarrow 0 \\
\text{for } i \in 0..N-1 \\
\left| \begin{array}{l}
h_i \leftarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\
\alpha_i \leftarrow \begin{pmatrix} 0 \\ 0 \text{ on error } \frac{1}{t_N} \end{pmatrix}
\end{array} \right. \\
\text{for } k \in 0..N-1 \\
\left| \begin{array}{l}
z_k \leftarrow h_0 \\
(\alpha_k)_1 \leftarrow (\alpha_k)_1 - \delta\alpha_k \\
\phi_k \leftarrow \text{if} \left( r_k = \infty, 0, \frac{n_{k+1} - n_k}{r_k} \right) \\
\alpha_{k+1} \leftarrow \frac{1}{n_{k+1}} \cdot (n_k \cdot \alpha_k + h_k \cdot \phi_k) \\
\text{for } i \in 0..1 \\
(z_k)_i \leftarrow \text{if} \left[ (\alpha_{k+1})_i \neq 0, \frac{(h_k)_i}{(\alpha_{k+1})_i}, \infty \right] \\
s'_k \leftarrow (z_k)_1 \\
h_{k+1} \leftarrow h_k - \alpha_{k+1} \cdot t_{k+1} \text{ if } k \neq N-1 \\
\Phi \leftarrow \Phi + \phi_k - \frac{t_k - d_k}{n_k} \cdot \Phi \cdot \phi_k \\
f_k \leftarrow \text{if} \left( \Phi \neq 0, \frac{n_{k+1}}{\Phi}, \infty \right) \\
d_{k+1} \leftarrow \text{if} \left[ \Phi \neq 0, f_k - (z_k)_0, 0 \right]
\end{array} \right. \\
F \leftarrow \frac{n_N}{\Phi} \\
B \leftarrow \text{Sy}_{0..N} \cdot \left| \frac{(z_{N-1})_0 - (z_{N-1})_1}{F} \right| \\
\begin{pmatrix} F \\ s' \\ B \end{pmatrix}
\end{array} \right.
\end{array}$$

.33.

*Pupil*

$$\text{Syst} := \begin{pmatrix} R_0 & R_1 & \dots & R_{N-1} & D \\ t_0 & t_1 & \dots & t_{N-1} & z_0 \\ n_0 & n_1 & \dots & n_{N-1} & n_N \\ \delta\alpha_0 & \delta\alpha_1 & \dots & \delta\alpha_{N-1} & 0 \end{pmatrix}$$

(34)  
Pupil

( . 33)

*D.*

$$\delta\alpha = \frac{\delta\tau}{D}. \tag{35}$$

Pupil

$$S' = \frac{S \cdot f'}{S + f'}, \tag{36}$$

*S' -*

Syst

100,

10,

20 .

1.5.

(36)

$$\text{Системная матрица } \text{Syst} := \begin{pmatrix} \infty & -100 & 20 \\ 0 & 10 & -400 \\ 1 & 1.5 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Фокусное расстояние  $f(r2,n) := \frac{-r2}{n-1}$

Передний кардинальный отрезок  $d(r2,n,t) := \frac{n-1}{n \cdot r2} \cdot t \cdot f(r2,n) \quad d(-100, 1.5, 10) = -6.667$

Передний отрезок  $S(r2,n,t) := -400 + d(r2,n,t) \quad S(-100, 1.5, 10) = -406.667$

Задний отрезок  $S'(S,f) := \frac{S \cdot f}{S + f}$

$A := \text{Pupil}(\text{Syst}) \quad A = \begin{pmatrix} 200 \\ 393.548 \\ 19.355 \end{pmatrix} \quad f(-100, 1.5) = 200$   
 $S'(S(-100, 1.5, 10), f(-100, 1.5)) = 393.548$

.34.

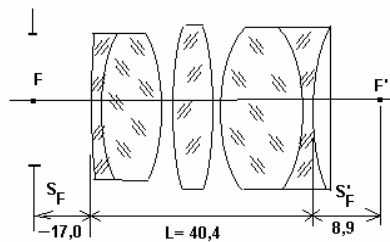
*MahtCad*

*Pupil.*

(

)

[3, .394] ( .35)



.35.

2.

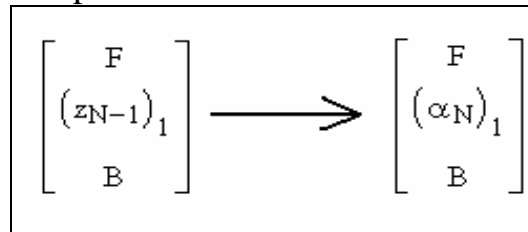
$r$	$d$	$N$		$D$
170,23	1,8	1,6199	( 13)	32,5
34,42				
	13,8	1,5163	( 8)	
-29,41				
	0,25	1		
70,78				
	7,6	1,5163	( 8)	36,5
-70,78				
	0,25	1		
31,89				
	15,0	1,5163	( 8)	
-31,89				36,5
	1,7	1,6199	( 13)	
56,01				

$$| \delta\alpha | < 0,01.$$

$$| \delta\alpha | < \frac{0,01}{36,5} 2 = 5,5 \cdot 10^{-4}.$$

$$\text{Syst1}(\tau) := \begin{pmatrix} 170.23 & 34.42 & -29.41 & 70.78 & -70.78 & 31.89 & -31.89 & 56.01 & 32.5 \\ 0 & 1.8 & 13.8 & 0.25 & 7.6 & 0.25 & 15.0 & 1.7 & -17.0 \\ 1 & 1.6199 & 1.5163 & 1 & 1.5163 & 1 & 1.5163 & 1.6199 & 1 \\ 0 & 0 & 0 & \frac{\tau}{18.25} & \frac{\tau}{18.25} & 0 & 0 & 0 & 0 \end{pmatrix}$$

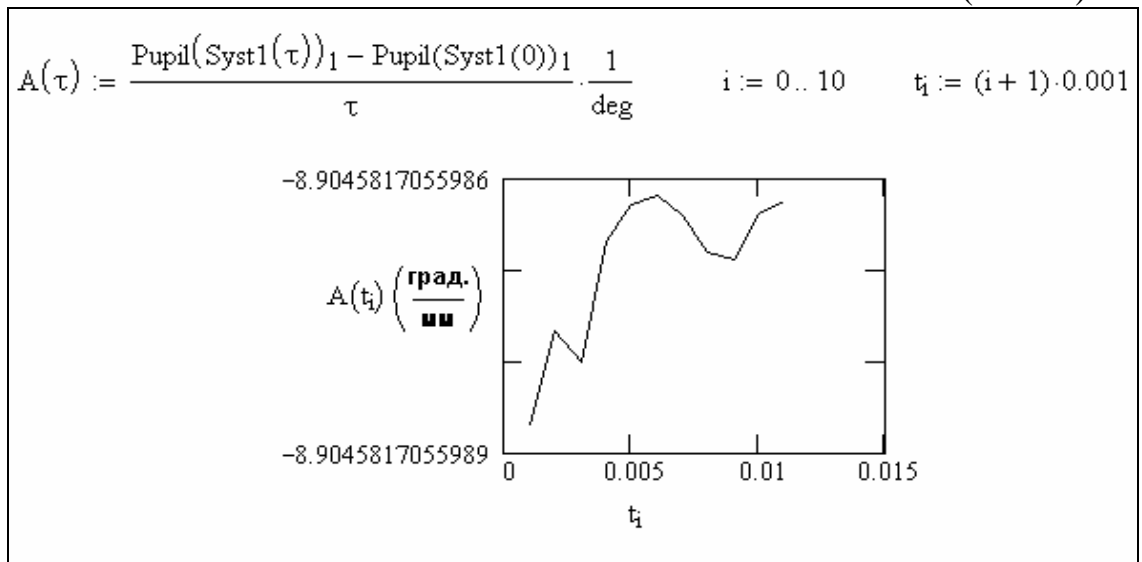
Pupil



.36.

Pupil.

( .37)



.37.

Pupil.

11

$$= -8,9 \quad / \quad 1 \quad 11$$

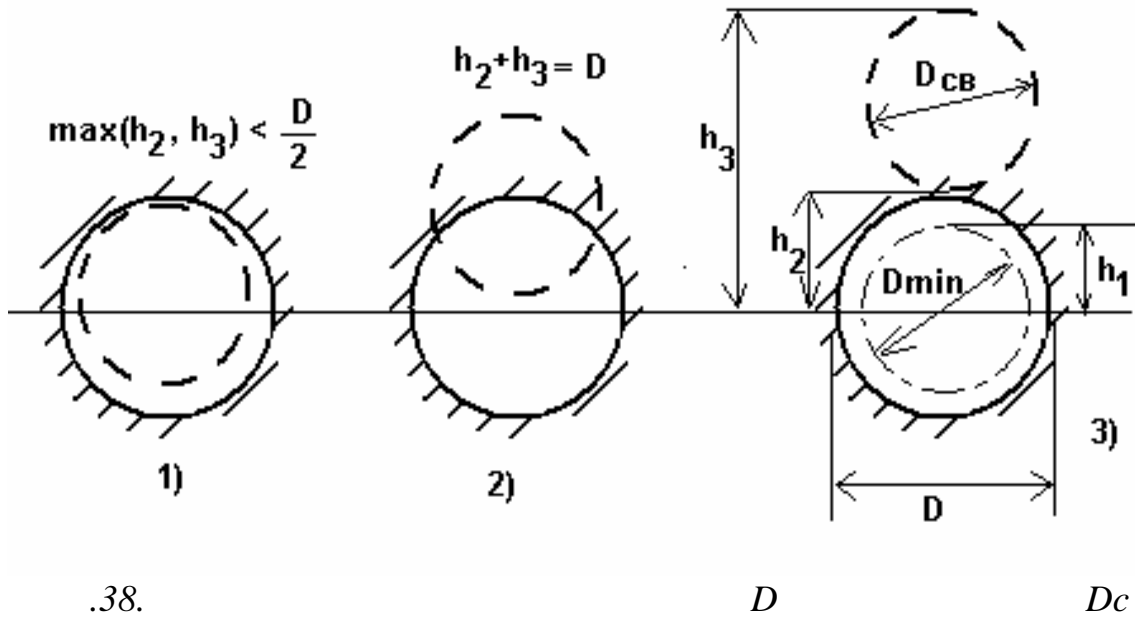
8.

( )

- 1)
- 2)
- 3)

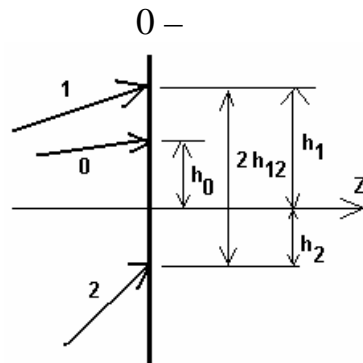
; 50%;

.38



.38.

.39



.39.

$$D_k = 2 \max(|h_1|, |h_2|).$$

(38)

50%,

$D_v$  ( .13)

$$D_k = 2 \cdot Dv(0.5, h_1, h_2). \quad (39)$$

$$D_k = 2 \cdot |h_0|, \quad (40)$$

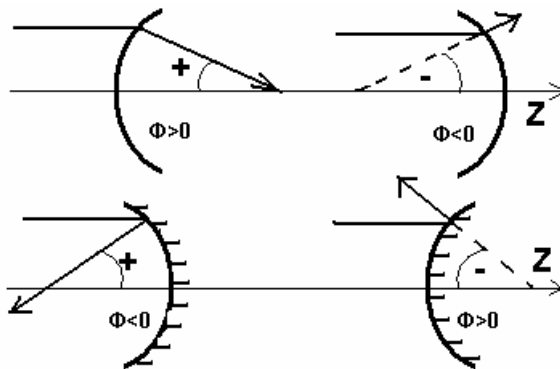
$$\alpha_{k+1} = \frac{1}{n_k} (n_{k-1} \alpha_k + h_k \Phi_k). \quad (41)$$

$k=0$

.43,

(  $<0$ )  $h_k > 0$   $n_k > 0$   $n_k < 0$ .

40,



.40.

$OYZ$

$OYZ$ .

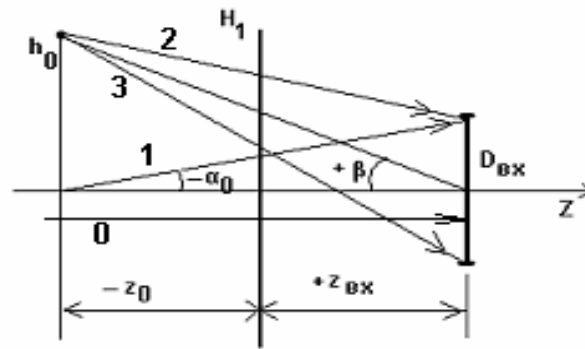
( .38).

$z$

$z_0$ ,

$D$

.41.



.41.

0

1 -

2 3

: 0% 50%.

50%

$$\text{Syst} = \begin{pmatrix} R_0 & R_1 \dots \infty & R_{k+1} \dots R_{N-1} & D_{BX} & \beta \\ t_0 & t_1 \dots t_{k-1} & t_k \dots t_{N-1} & z_0 & z_{BX} \\ n_0 & n_1 \dots 1 & 1 \dots n_{N-1} & n_N & k \\ D_0 & D_1 \dots D_{k-1} & D_k \dots D_{N-1} & V & 0 \end{pmatrix} \quad (42)$$

: D -

, z\_0 -

(

), n\_N -

V, V = 0,



,  $V = 1$ ,  
 : - ( ),  $z -$ ,  $k -$   
 ,  
 ( ),

- . 42,  
 . 43.

- 1) , ,
- 2) , ,
- 3) .  
 deg,  
 5 · deg.

1.  $V$ ,  $D \leftarrow ((Syst)^T)^{(3)}$ .

$$\gamma = \text{tg}(\alpha_0) = \frac{D}{2(z_0 - z_1)}, z_0 \neq \infty, z_1 \neq \infty. \quad (43)$$

2. 4-

$$\alpha_0 = \frac{\gamma}{\text{tg}(\beta) + \gamma}, \quad (44)$$

$$3. : h_0 = \frac{D}{2} \frac{1}{1 - z_1 \alpha_0}, z_1 \neq \infty, \quad (45)$$

4. Vign  
 $V,$   
 $h,$   $V$   
 $: H_0 \leftarrow Vign(h_0, V, D).$   
 $H_0$

5.  $(V=1),$  3-

6. Pupil.  
 4-

$h_0$  0,  
 $z_k \leftarrow h_0,$   $k-$

7. Pupil.

8. Pupil.

9.

MathCAD

$$a = \frac{2}{8}, \quad b = \frac{1}{4}$$

$$a \cdot b = 2 \cdot 1 + 8 \cdot 4 = 34$$

$$\vec{(a \cdot b)} = \begin{pmatrix} 2 \\ 32 \end{pmatrix} \quad \vec{\frac{a}{b}} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$i = 0 \dots 3$$

if,

$$(z_k)_i \leftarrow \text{if} \left[ (\alpha_{k+1})_i \neq 0, \frac{(h_k)_i}{(\alpha_{k+1})_i}, \infty \right]$$

10.

( Pupil),

: if  $k \neq N - 1,$   $k -$

,  $N -$

ORIGIN=0,

“ ”  
“ ”

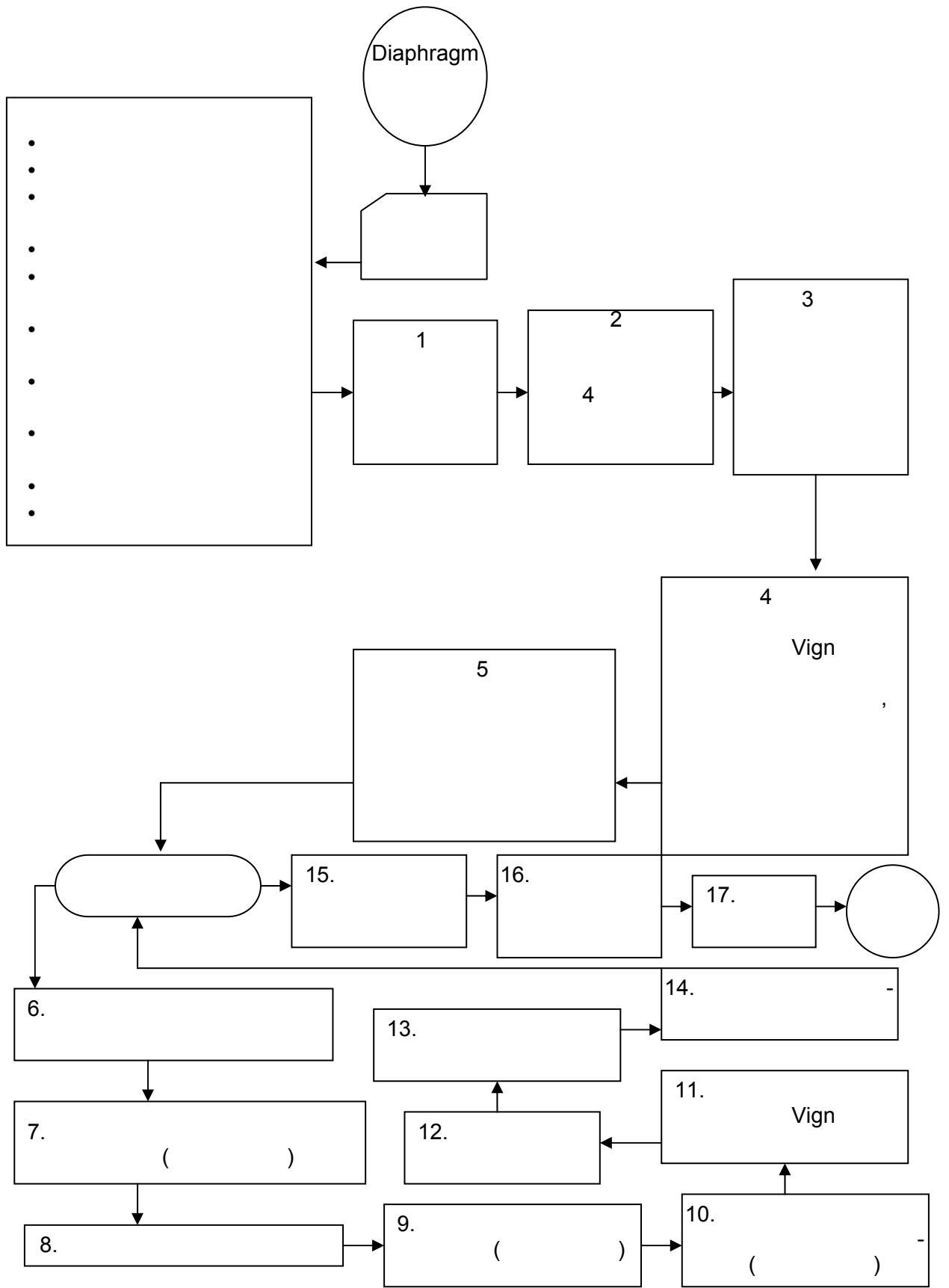
“OK”.

11.

Vign

12.

Pupil.



Diaphragm(Syst) :=	$r \leftarrow ((\text{Syst})^T)^{\langle 0 \rangle}$ $t \leftarrow ((\text{Syst})^T)^{\langle 1 \rangle}$ $n \leftarrow ((\text{Syst})^T)^{\langle 2 \rangle}$ $D \leftarrow ((\text{Syst})^T)^{\langle 3 \rangle}$ $N \leftarrow \text{cols}(\text{Syst}) - 2$ $v \leftarrow \text{Syst}_{3, N}$ $R_v \leftarrow \frac{r_N}{2}$ $\beta \leftarrow \tan(r_{N+1})$ $z_v \leftarrow t_{N+1}$ $z_0 \leftarrow t_N$ $\gamma \leftarrow \text{if} \left( z_0 \neq \infty \wedge z_v \neq \infty, \frac{R_v}{z_0 - z_v}, 0 \right)$ $\alpha_0 \leftarrow (0 \ \gamma \ \beta + \gamma \ \beta - \gamma)^T$	<b>Извлечение информации из матрицы Syst</b> <b>радиусы</b> <b>толщины</b> <b>показатели преломления</b> <b>диаметры диафрагм</b> <b>число поверхностей</b> <b>признак</b> ( $v=0$ вычисление $D_0, D_{1/2}, D_{\min}$ $v=1$ вычисление коэффициентов виньетирования) <b>радиус входного зрачка</b> <b>полевой угол (рад)</b> <b>расстояние от осевой точки первой</b> <b>поверхности до входного зрачка</b> <b>расстояние до плоскости предмета</b> <b>вычисление тангенса</b> <b>переднего апертурного</b> <b>угла</b> <b>вектор тангенсов углов входных лучей</b>
--------------------	---	--

$h_0 \leftarrow R_v \cdot (1 \ 1 \ 1 \ -1)^T + \text{if} (z_v \neq 0, z_v \cdot \alpha_0, 0)$ $di_0 \leftarrow \text{Vign}(h_0, v, D_0)$ $\Phi \leftarrow 0$ $d_0 \leftarrow 0$ <b>for</b> $k \in 0..N-1$ $z_k \leftarrow h_0$ $\phi_k \leftarrow \text{if} \left( r_k = \infty, 0, \frac{n_{k+1} - n_k}{r_k} \right)$ $\alpha_{k+1} \leftarrow \frac{1}{n_{k+1}} \cdot (n_k \cdot \alpha_k + h_k \cdot \phi_k)$ <b>for</b> $i \in 0..3$ $(z_k)_i \leftarrow \text{if} \left[ (\alpha_{k+1})_i \neq 0, \frac{(h_k)_i}{(\alpha_{k+1})_i}, \infty \right]$	<b>вектор высот лучей</b> <b>Обращение к программе Vign</b> <b>Счётчик оптической силы</b> <b>Задний кардинальный отрезок первой поверхности</b> <b>заголовок цикла по числу поверхностей</b> <b>устанавливаем шаблон вектора</b> <b>отрезков по образцу вектора высот</b> <b>оптическая сила к-поверхности</b> <b>вычисление вектора</b> <b>преломленных лучей</b> <b>цикл по элементам вектора задних отрезков</b> <b>вычисление</b> <b>к-того заднего отрезка</b>
---	--

.43.

*Diaphragm*

if $k \neq N - 1$ так как для последней поверхности нет последующей	
$h_{k+1} \leftarrow h_k - \alpha_{k+1} \cdot t_{k+1}$	вычисление высот на последующей поверхности
$d_{k+1} \leftarrow \text{Vign}(h_{k+1}, v, D_{k+1})$	обращение к программе Vign
$\Phi \leftarrow \Phi + \phi_k - \frac{t_k - d_k}{n_k} \cdot \Phi \cdot \phi_k$	вычисление оптической силы текущей части системы
$f_k \leftarrow \text{if} \left( \Phi \neq 0, \frac{n_{k+1}}{\Phi}, \infty \right)$	текущее фокусное расстояние
$d_{k+1} \leftarrow \text{if} \left[ \Phi \neq 0, f_k - (z_k)_0, 0 \right]$	задний кардинальный отрезок текущей части оптической системы
цикл завершён	
$v \leftarrow \frac{(z_{N-1})_0 - (z_{N-1})_1}{f_{N-1}}$	линейное увеличение системы
$Y' \leftarrow \text{if}(z_0 \neq \infty, z_0 \cdot \gamma \cdot v, 0)$	линейный размер предмета
$\left[ f_{N-1} \quad (z_{N-1})_1 \quad Y' \quad d_i \right]^T$	Вектор выводимых параметров

.43 .

*Diaphragm*

13.

$$f_k \leftarrow \text{if} \left( \Phi \neq 0, \frac{n_{k+1}}{\Phi}, \infty \right)$$

14.

$$d_{k+1} \leftarrow \text{if} \left[ \Phi \neq 0, f_k - (z_k)_0, 0 \right]$$

$d_N$

15.

$$v = \frac{-x'}{f'}, \quad x' -$$

$$v \leftarrow \frac{(z_{N-1})_0 - (z_{N-1})_1}{f_{N-1}}$$

16.

$z_0$

$$Y' \leftarrow \text{if}(z_0 \neq \infty, z_0 \cdot \gamma \cdot v, 0)$$

17.

$f_{N-1};$

$Y;$

$(z_{N-1})_1;$

8.1.

Vign

)

$V=0,$

$D_0 = 2 \max(|h_2|, |h_3|)$  (46)

50%:

$D_1 = Dv(0.5, h_1, h_2)$  (47)

$D_2 = 2 \cdot |h_1|.$  (48)

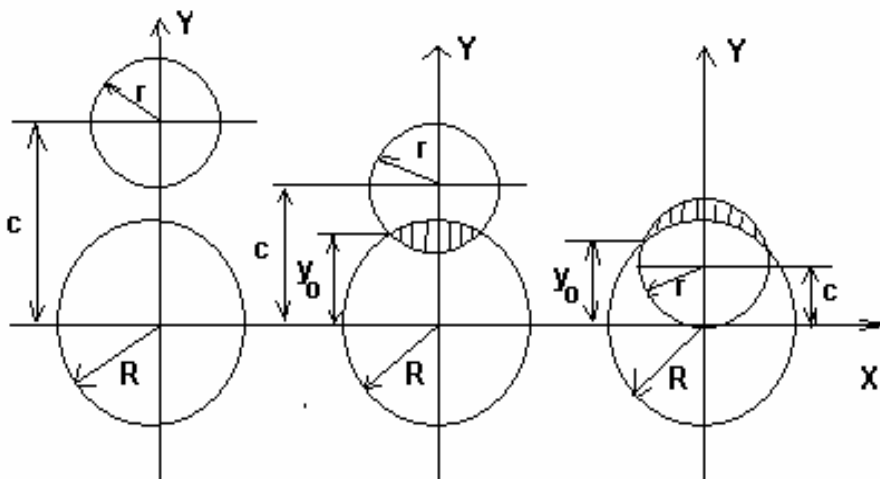
)

$V = 1,$

$\eta = \frac{S_o}{\pi \rho^2}.$  (49)

$S_0$

( .44)



.44.

1)  $c > r+R - S_o=0,$

2)  $c > y_0 -$

$, S_o > 0,$

3)  $c < y_0 -$   
 $r^2 - S_v.$

$S_v$

$S_o =$

$x_0$

:



$$S_0(c > y_0) = 2 \int_0^{x_0} [\sqrt{R^2 - x^2} - (c - \sqrt{r^2 - x^2})] dx = I(R, x_0) + I(r, x_0) - 2cx_0$$

$$S_0(c \leq y_0) = \pi r^2 - 2 \int_0^{x_0} [(c + \sqrt{r^2 - x^2}) - \sqrt{R^2 - x^2}] dx = \pi r^2 + I(R, x_0) - I(r, x_0) - 2cx_0, \quad (50)$$

$$I(A, x) = 2 \int_0^x \sqrt{A^2 - t^2} dt = x\sqrt{A^2 - x^2} + A^2 \arcsin(x/A)$$

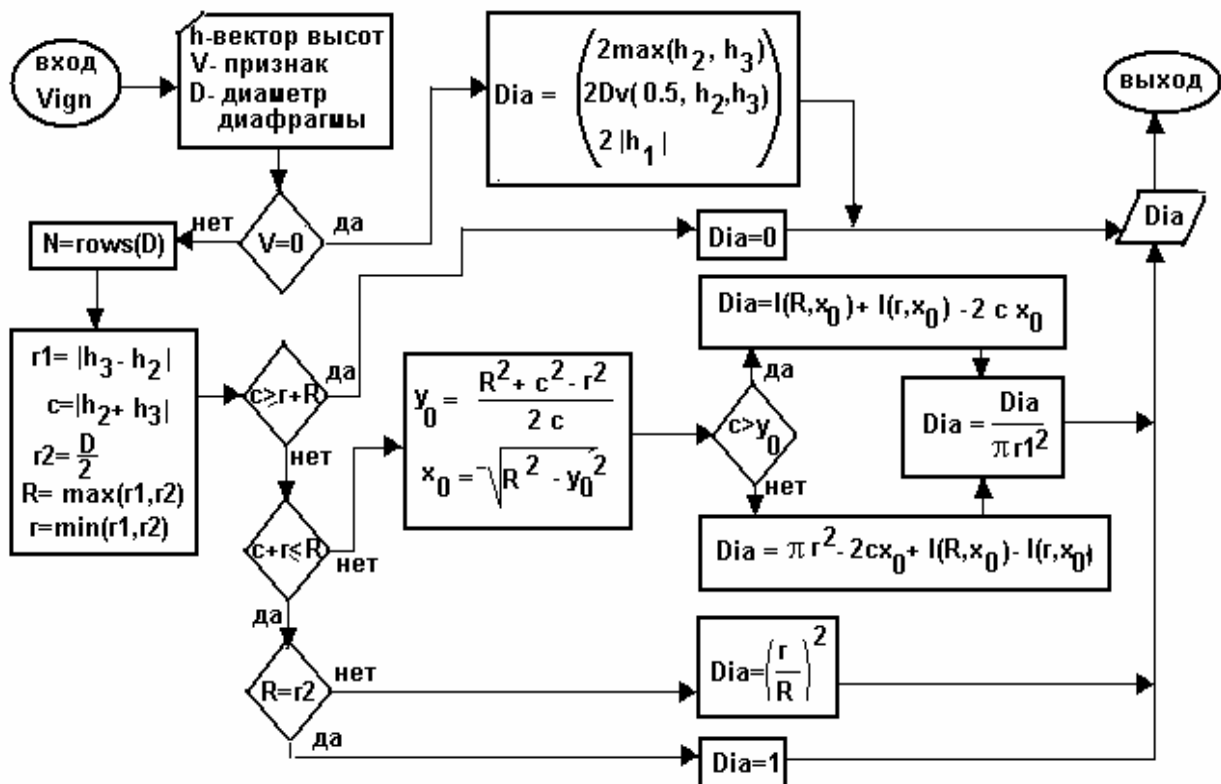
$$\begin{aligned} x^2 + y^2 &= R^2 \\ x^2 + (y-c)^2 &= r^2 \end{aligned} \Rightarrow y_0 = \frac{R^2 + c^2 - r^2}{2c}, \quad x_0 = \sqrt{R^2 - y_0^2} \quad (51)$$

$$r_1 = 0,5|h_3 - h_2|.$$

$$c = |h_2 + h_3|.$$

$$R = \max(r_1, D/2),$$

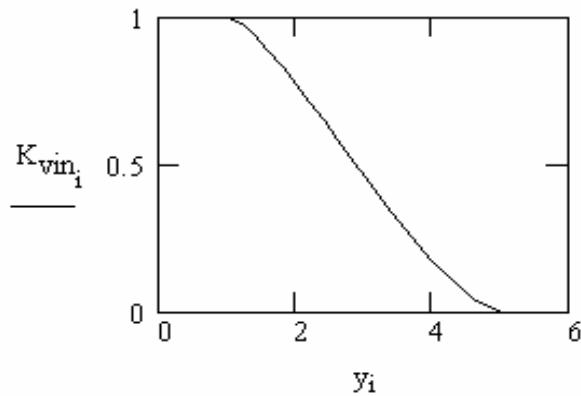
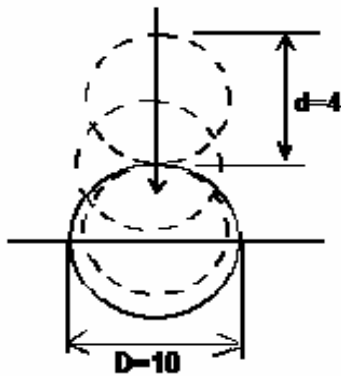
$$r = \min(r_1, D/2), \quad D -$$



.45.

(.47).

$$f(t) := \text{Vign} \left[ \begin{pmatrix} 5 \\ 0 \\ t \\ t+4 \end{pmatrix}, 1, 10 \right] \quad i := 0..20 \quad y_i := 5 - \frac{i}{5} \quad K_{\text{vign}_i} := f(y_i)$$



.46.

*MathCAD,*

*Vign*

$$\text{Vign}(h, V, D) := \begin{cases} \text{Dia} \left\langle \begin{pmatrix} 2 \cdot \max(h_2, h_3) \\ 2 \cdot \text{Dv}(0.5, h_2, h_3) \\ 2 \cdot |h_1| \end{pmatrix} \right\rangle & \text{if } V = 0 \\ \text{otherwise} \\ \begin{cases} r1 \leftarrow 0.5 \cdot |h_3 - h_2| \\ c \leftarrow 0.5 \cdot |h_2 + h_3| \\ r2 \leftarrow 0.5 \cdot D \\ R \leftarrow \max(r1, r2) \\ r \leftarrow \min(r1, r2) \\ \text{return } 0 & \text{if } c \geq r + R \\ \text{if } c + r \leq R \\ \begin{cases} \text{return } 1 & \text{if } R = r2 \\ \left( \frac{r}{R} \right)^2 & \text{otherwise} \end{cases} \\ \text{otherwise} \\ \begin{cases} y0 \leftarrow \frac{R^2 + c^2 - r^2}{2 \cdot c} \\ x0 \leftarrow \sqrt{R^2 - y0^2} \\ \text{return } \frac{I(R, x0) + I(r, x0) - 2 \cdot c \cdot x0}{\pi \cdot r1^2} & \text{if } c > y0 \\ \text{return } \frac{\pi \cdot r^2 - 2 \cdot c \cdot x0 + I(R, x0) - I(r, x0)}{\pi \cdot r1^2} & \text{otherwise} \end{cases} \end{cases} \end{cases}$$

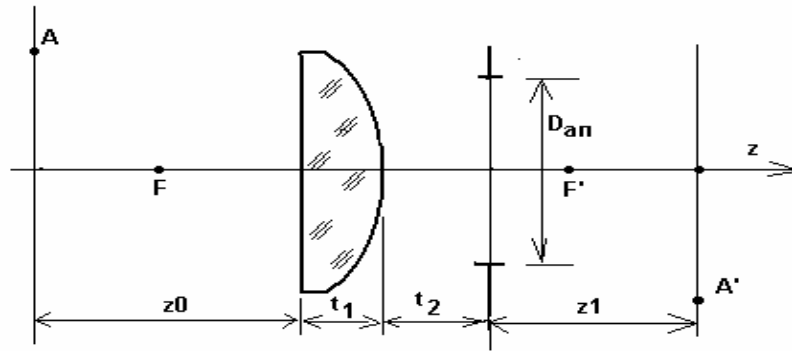
.47.

Vign

8.2.

( )

( .48).



.48.

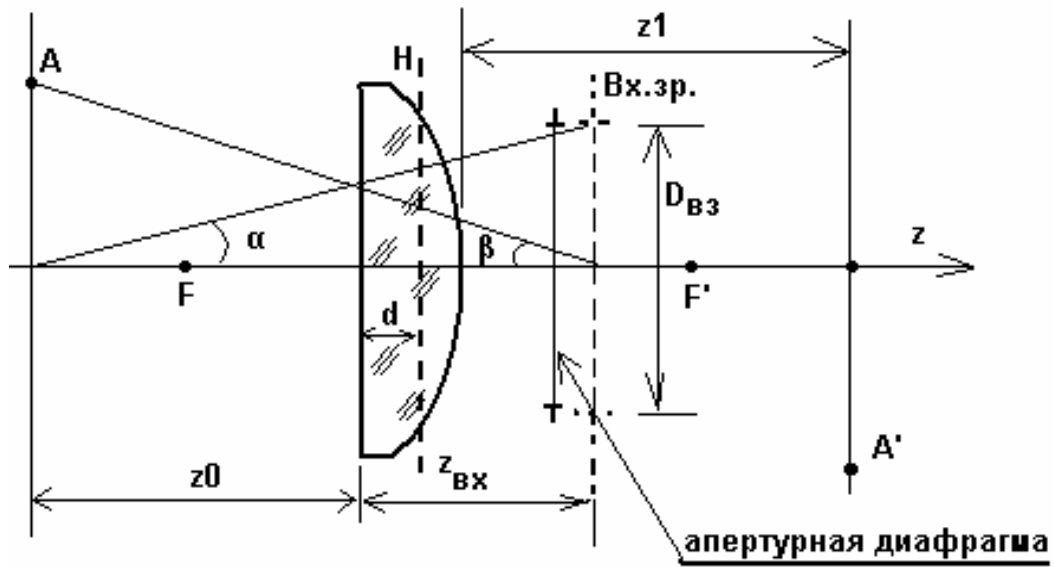
$$R_1 = \infty, R_2 = -100, t_1 = 10, t_2 = 15, D = 20, n = 1,5$$

$$\text{Syst} := \begin{pmatrix} 100 & \infty & 20 \\ 0 & 10 & -15 \\ 1 & 1.5 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Pupil,

$$\text{Pupil}(\text{Syst}) = \begin{pmatrix} 200 \\ -22.883 \\ 21.622 \end{pmatrix} \begin{array}{l} \text{фокусное расстояние} \\ \text{задний отрезок (расстояние до входного зрачка)} \\ \text{диаметр входного зрачка} \end{array}$$

( .49).



.49.

0 50%

A,  
= 45°.

$$\text{Syst} := \begin{pmatrix} \infty & -100 & \infty & 21.622 & 45 \cdot \text{deg} \\ 0 & 10 & 15 & -393.333 & 22.883 \\ 1 & 1.5 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$z_0 = -(2f' - d).$$

Syst0.

MathCAD:

$$\text{Syst0} := \begin{pmatrix} 100 & \infty & 20 \\ 0 & 10 & \infty \\ 1 & 1.5 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$B := \text{Pupil}(\text{Syst0}) \quad B = \begin{pmatrix} 200 \\ 193.333 \\ 0 \end{pmatrix}$$

$$d := B_0 - B_1 \quad d = 6.667$$

Diaphragm

$$s := \text{Diaphragm}(\text{Syst}) \quad s = \begin{pmatrix} 200 \\ 385 \\ -1 \\ \{3,1\} \end{pmatrix} \begin{array}{l} \text{фокусное расстояние} \\ \text{Последний отрезок (до} \\ \text{апертурной диафрагмы)} \\ \text{линейное увеличение} \\ \text{вектор диафрагм} \end{array}$$

поверхность 1	поверхность 2	апертурная диафрагма	Виньетирование
$(s_3)_0 = \begin{pmatrix} 66.199 \\ 45.766 \\ 20.433 \end{pmatrix}$	$(s_3)_1 = \begin{pmatrix} 53.212 \\ 32.433 \\ 20.78 \end{pmatrix}$	$(s_3)_2 = \begin{pmatrix} 20.001 \\ 20 \\ 20 \end{pmatrix}$	0%
			50%
			Dmin
			50%-

$$\text{Syst} := \begin{pmatrix} \infty & -100 & \infty & 21.622 & 45 \cdot \text{deg} \\ 0 & 10 & 15 & -393.333 & 22.883 \\ 1 & 1.5 & 1 & 1 & 2 \\ 45.766 & 32.433 & 20 & 1 & 0 \end{pmatrix} \begin{array}{l} \text{Diaphragm} \\ \text{Diaphragm} \end{array}$$

$v = 1,$

$$s := \text{Diaphragm}(\text{Syst}) \quad s = \begin{pmatrix} 200 \\ 385 \\ -1 \\ \{3,1\} \end{pmatrix}$$

**Коэффициенты виньетирования**

$$(s_3)_0 = 0.452 \quad (s_3)_1 = 0.431$$

50%

9.

$(x, y, z)$

$(p, q, m).$

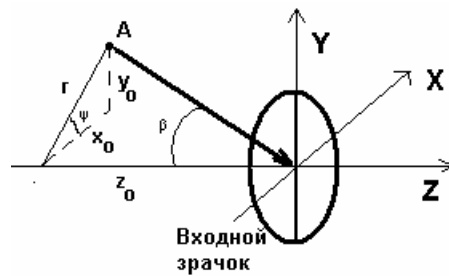
$$p = \cos(\alpha), \quad q = \cos(\beta), \quad m = \cos(\gamma), \quad (52)$$

$$p^2 + q^2 + m^2 = 1. \quad (53)$$

$$\gamma \cdot n' - \gamma \cdot n = h \cdot \varphi. \quad (54)$$

OZ

(.50).



.50.

$$\beta = \text{arctg} \frac{r}{z_0} = \text{arctg} \frac{\sqrt{x_0^2 + y_0^2}}{z_0}. \quad (55)$$

$$\psi = \text{arctg} \frac{y_0}{x_0}. \quad (56)$$

(.51)  
OYZ

( OZ)

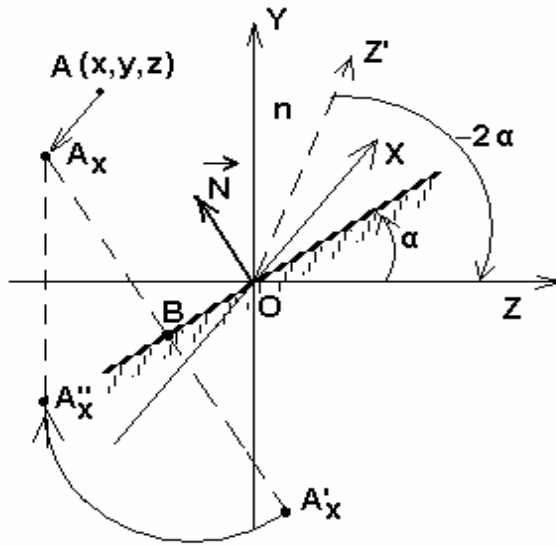
N ( )

2 .

2

OX

(.51)



.51.

$(x_0, y_0, z_0)$   
A' A''

$(0, q, m)$ .

A A'.

$$B = \frac{A + A'}{2} \rightarrow A' = 2B - A. \quad (57)$$

$$q = n \cdot \cos(\alpha), \quad m = n \cdot \sin(\alpha). \quad (58)$$

$q > 0$ .

OYZ:

$$y = -z \cdot \text{tg}(\alpha) \rightarrow y \cdot \cos(\alpha) + z \cdot \sin(\alpha) = 0. \quad (59)$$

,  $A(y_0, z_0), B(y_1, z_1)$ ,

$$\frac{\cos(\alpha)}{y_1 - y_0} = \frac{\sin(\alpha)}{z_1 - z_0} \rightarrow y_1 \sin(\alpha) - z_1 \cos(\alpha) = y_0 \sin(\alpha) - z_0 \cos(\alpha). \quad (60)$$

(59), (60)

$$\begin{matrix} y_1 & z_1, \\ y_1 & = \frac{\sin(\alpha)}{\cos(\alpha)} (-y_0 \sin(\alpha) + z_0 \cos(\alpha)). \end{matrix} \quad (61)$$

A':

$$\begin{aligned} y' &= 2y_1 - y_0 = -y_0 \cos(2\alpha) - z_0 \sin(2\alpha) \\ z' &= 2z_1 - z_0 = -y_0 \sin(2\alpha) + z_0 \cos(2\alpha) \end{aligned} \quad (62)$$

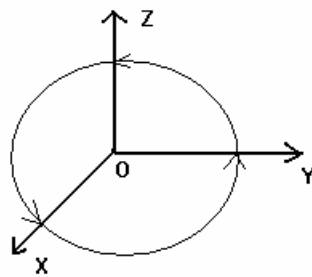
$$\begin{aligned}
 & \text{, } x' = x_0 \\
 & \text{:} \\
 & \begin{matrix} x' & x_0 & & 1 & 0 & 0 \\ y' & = M_1 y_0 & , & M_1 = & 0 & -\cos(2\alpha) & -\sin(2\alpha) \\ z' & z_0 & & 0 & -\sin(2\alpha) & \cos(2\alpha) \end{matrix} \quad (63) \\
 & \text{-1.}
 \end{aligned}$$

2

*OX*

-2 .

52



.52.

1)

2)

$$: y' = y \cdot \cos(\alpha) + z \cdot \sin(\alpha),$$

3)

$$z' = z \cdot \cos(\alpha) - y \cdot \sin(\alpha).$$

*OX*

-2

$$\begin{matrix} 1 & 0 & 0 \\ M_0 = & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ & 0 & \sin(2\alpha) & \cos(2\alpha) \end{matrix} \quad (64)$$

1.

0



$$A'' = M_0 A' = M_0 M_1 A, \quad M = M_0 M_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (65)$$

$OX$ ,

$(p, q, m)$ ,

$(x_0, y_0, z_0)$ .

$Z$ .

1)

$OX$

$OYZ$ .

2)

(65).

3)

1.

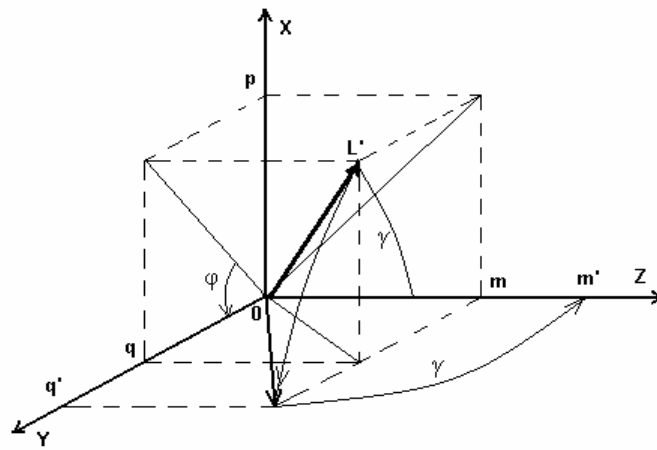
$OYZ$

: .53.

$$\sin(\varphi) = \frac{p}{\sqrt{p^2 + q^2}}, \quad \cos(\varphi) = \frac{q}{\sqrt{p^2 + q^2}}. \quad (66)$$

$OX$  ( )  $OY$  ( )  $OZ$  ( ) .

$$S = \begin{pmatrix} \cos(\varphi) & \sin(\varphi) & 0 \\ -\sin(\varphi) & \cos(\varphi) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{1}{\sqrt{p^2 + q^2}} \begin{pmatrix} q & p & 0 \\ -p & q & 0 \\ 0 & 0 & \sqrt{p^2 + q^2} \end{pmatrix}. \quad (67)$$



.53.

OX

$$S^{-1} = \frac{1}{\sqrt{p^2 + q^2}} \begin{pmatrix} q & -p & 0 \\ p & q & 0 \\ 0 & 0 & \sqrt{p^2 + q^2} \end{pmatrix} \quad (68)$$

$$T = S^{-1}M \cdot S = \frac{1}{p^2 + q^2} \begin{pmatrix} q^2 - p^2 & 2pq & 0 \\ 2pq & p^2 - q^2 & 0 \\ 0 & 0 & \sqrt{p^2 + q^2} \end{pmatrix} \quad (69)$$

(69)

$$T \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}$$

T

Pupil

9.1.

MathCAD

$$\text{Info} := \begin{pmatrix} z'_0 & z'_1 & \dots & z'_{N-1} & D_v \\ t_0 & t_1 & \dots & t_{N-1} & z_v \\ P_0 & P_1 & \dots & P_{N-1} & z_0 \\ D_0 & D_1 & \dots & D_{N-1} & 1 \vee 0 \\ \delta\alpha_0 & \delta\alpha_1 & \dots & \delta\alpha_{N-1} & \beta \\ \delta\rho_0 & \delta\rho_1 & \dots & \delta\rho_{N-1} & \psi \end{pmatrix} \quad (70)$$

1.  $k$  ( $k=0,1,\dots, N-1$ ).

2. ( $t_0=0$ ).

3.  $p + i q$ ,  $p, q$  — X- Y-

4.  $D$ ,  $D+i$ ,  $i$  —

5.

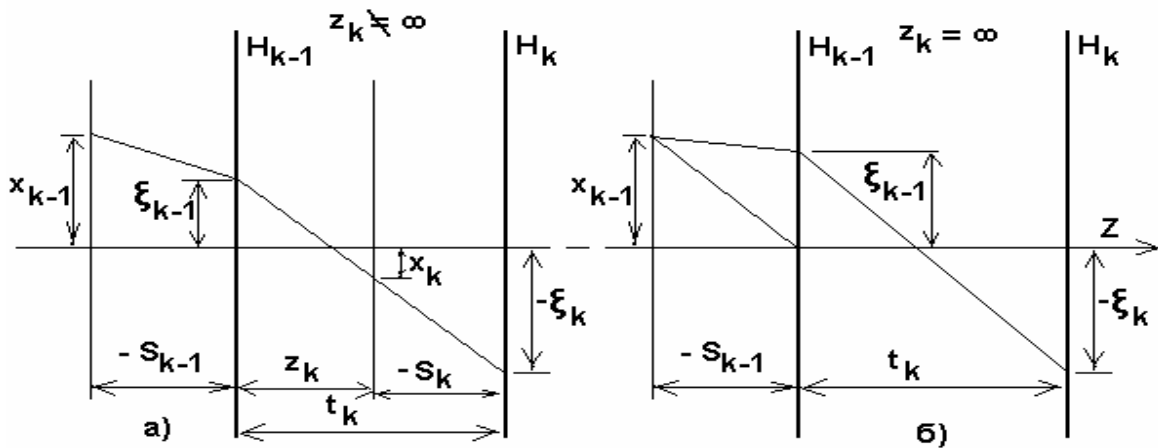
6.

Info:

- 
- 
- 
- 
- 
- 

( )

.54.



.54.

$S_{k-1} \neq \infty,$   
 $H_{k-1}$

$(x_{k-1}, y_{k-1})$   
 $z_k \neq \infty,$

$$V_{k-1} = \frac{z_k}{z_{k-1} - t_{k-1}}; \quad (71)$$

$$V_{k-1} = 1; \quad (71)$$

$$y_k): \quad x_k = V_{k-1} x_{k-1}, \quad y_k = V_{k-1} y_{k-1}. \quad (72)$$

$H_{k-1} -$   
( .50 )

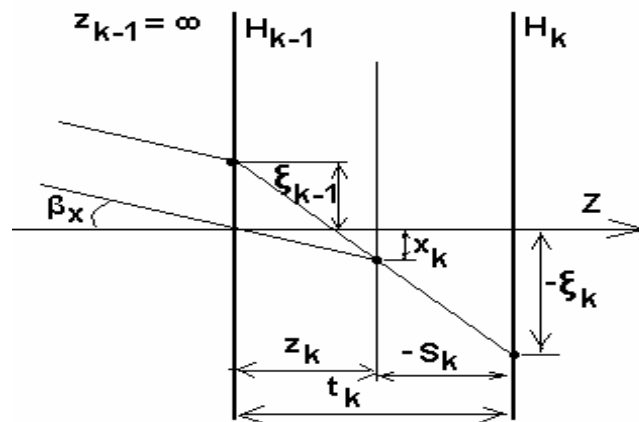
$$\begin{cases} \xi_k = \xi_{k-1} + \frac{t_k}{z_k} (x_k - \xi_{k-1}), \\ \eta_k = \eta_{k-1} + \frac{t_k}{z_k} (y_k - \eta_{k-1}) \end{cases} \quad (73)$$

$z_k = \infty$  ( . 50, ),

$$\begin{cases} \xi_k = \xi_{k-1} + \frac{t_k}{z_{k-1} - t_{k-1}} x_{k-1}, \\ \eta_k = \eta_{k-1} + \frac{t_k}{z_{k-1} - t_{k-1}} y_{k-1} \end{cases} \quad (74)$$

$$\text{tg}(\beta_{k-1}) = \frac{\sqrt{x_{k-1}^2 + y_{k-1}^2}}{|z_{k-1} - t_{k-1}|}, \quad \text{tg}(\psi) = \frac{y_{k-1}}{x_{k-1}} \quad (75)$$

$z_{k-1} = \infty$ , ( . 55),



.55.

$$x_k = z_k \text{tg}(\beta_{k-1}) \cos(\psi_{k-1}), \quad y_k = z_k \text{tg}(\beta_{k-1}) \sin(\psi_{k-1}). \quad (76)$$

$$(k, k) \quad (73).$$

$$x = (z_V - z_0) \text{tg}(\ ) \cos(\ ), \quad y = (z_V - z_0) \text{tg}(\ ) \sin(\psi). \quad (77)$$

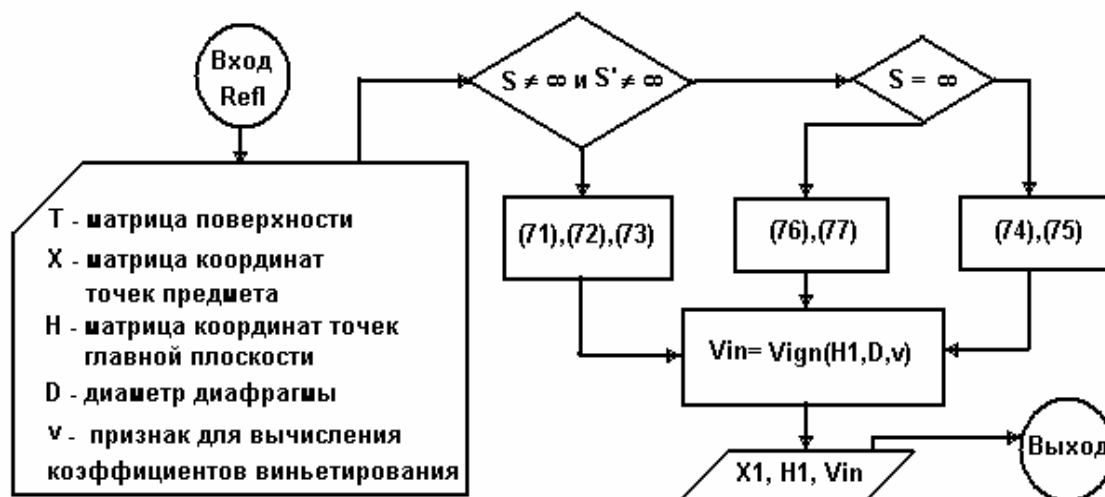
MathCAD

2 3.

$$X = \begin{pmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \end{pmatrix} \cdot \begin{matrix} (x_0, y_0), (x_1, y_1), (x_2, y_2) \end{matrix} \quad (78)$$

$$X = \begin{pmatrix} \text{tg}(u_0) & \text{tg}(u_1) & \text{tg}(u_2) \\ \Psi_0 & \Psi_1 & \Psi_2 \end{pmatrix} \quad (78)$$

$$P = \begin{pmatrix} t_0 & t_1 \\ S & S' \end{pmatrix} \quad (79)$$



.56.  
Refl

Refl

```

Refl(T,X,H,P,D,v) := if T1,0 ≠ ∞ ∧ T1,1 ≠ ∞
  | x1 ← if (Im(D) = 0, T1,1/T1,0, 1) · X
  | ξ1 ← H + T0,1/T1,1 · (x1 - H)
  otherwise
  | if T1,0 = ∞
  | | x1 ← T1,1 · stack( [ (X^T)^{<0>} · cos[(X^T)^{<1>}] , [ (X^T)^{<0>} · sin[(X^T)^{<1>}] ] )
  | | ξ1 ← H + T0,1/T1,1 · (x1 - H)
  otherwise
  | a ← augment(|X^{<0>}|, |X^{<1>}|, |X^{<2>}|)
  | for k ∈ 0..2
  | | ψk ← if(X1,k > 0, π, -π) on error atan(X1,k/X0,k) + if(X0,k > 0, 0, π)
  | x1 ← stack[a, (ψ0 ψ1 ψ2)]
  | ξ1 ← H + T0,1 · X / (T1,0 - T0,0)
  Vin ← Vign(ξ1, v, P, Re(D))
  (x1 ξ1 Vin)^T

```

.57.

“ ”

- Refl.**
- 1)  $T, X$  — .
- 2)  $-X$   $Y$   $p + i$
- 3)  $q$   $x1$  if,
- 4)  $k$  ,  $k$  — .
- 5) .
- 6) stack .
- 7) augment .

8) on error ,  
 9)  $x1,$   
 (v=0), (v=1).

Enter,

( , )

0 50% ,

```

Enter(Syst) := N ← cols(Syst) - 1
if Syst2,N ≠ ∞
  x ← (Syst1,N - Syst2,N) · tan(Syst4,N) · cos(Syst5,N)
  y ← (Syst1,N - Syst2,N) · tan(Syst4,N) · sin(Syst5,N)
  X ← ( 0 x x
        0 y y )
  X ← ( tan(Syst4,N) tan(Syst4,N) tan(Syst4,N) ) otherwise
        Syst5,N Syst5,N Syst5,N
  Y ← ( 0 0 0
        Syst0,N Syst0,N -Syst0,N )
  f ← stack( [ [ ((X)ᵀ)⁽⁰⁾ · cos[ ((X)ᵀ)⁽¹⁾ ] ], [ ((X)ᵀ)⁽⁰⁾ · sin[ ((X)ᵀ)⁽¹⁾ ] ] ] )
  H ← Y + Syst1,N if [ Syst2,N ≠ ∞, (X - Y) / (Syst1,N - Syst2,N), f ]
  Vin ← Vign(H, Syst3,N, Syst2,0, Re(Syst3,0))
  (X H Vin)ᵀ

```

.58. Enter

**Enter.**

1)

2)

3)



- 4)  $Y$  ,  $OYZ.$
- 5)  $f$  if, (76).  
 $f$  Refl.
- 6) Vign. ModelMir ( .62) Refl  
 Enter
- 1) **ModelMir**  $N$  (k=0..N-1)  
 Enter,
- 2) .
- 3) -
- 4) .
- 5) .
- 6)  $t1.$   $m.$
- 7)  $a$  Refl.
- 8)  $N$  (k=0..N-1)  
 Enter,
- 9) .
- 10) -

11)

```

ModelMir(Syst) :=
  A ← Enter(Syst)
  X ← A0
  H ← A1
  Vin0 ← A2
  N ← cols(Syst) - 1
  for k ∈ 0..N - 1
    z0k ← if(k = 0, Syst2, N, Syst0, k-1 - Syst1, k)
    q ← if(|Syst4, k| = 0, 0, arg(Syst4, k))
    for o ∈ 0..2
      if z0 ≠ ∞
        r ← √((|X<sup>0</sup>|)² + (z0k)²)
        X0, o ← X0, o - r·tan(|Syst4, k|)·cos(q) - Re(Syst5, k)
        X1, o ← X1, o - r·tan(|Syst4, k|)·sin(q) - Im(Syst5, k)
      for o ∈ 1..2
        otherwise
          X0, o ← tan(atan(X0, o) - |Syst4, k|)
          X1, o ← X1, o - q
    t1 ← if(k ≠ N - 1, Syst1, k+1, Syst0, k)
    m ←  $\begin{matrix} \text{Syst1, k} & t1 \\ z0k & \text{Syst0, k} \end{matrix}$ 
    a ← Refl(m, X, H, Syst2, k+1, Syst3, k+1, Syst3, N)
    X ← a0
    H ← a1
    Vin<sub>k+1</sub> ← a2
  X<sup>1</sup>
  Vin

```

12)

.59. *ModelMir*  
 N (k=0..N-1)  
 Enter,

13)

14)

15)

16)

$t_1$ .

17)

$m$ .

18)

$a$

Refl.

19)

(

$k=N-1$ )

$X$   $H$ ,

Vign,

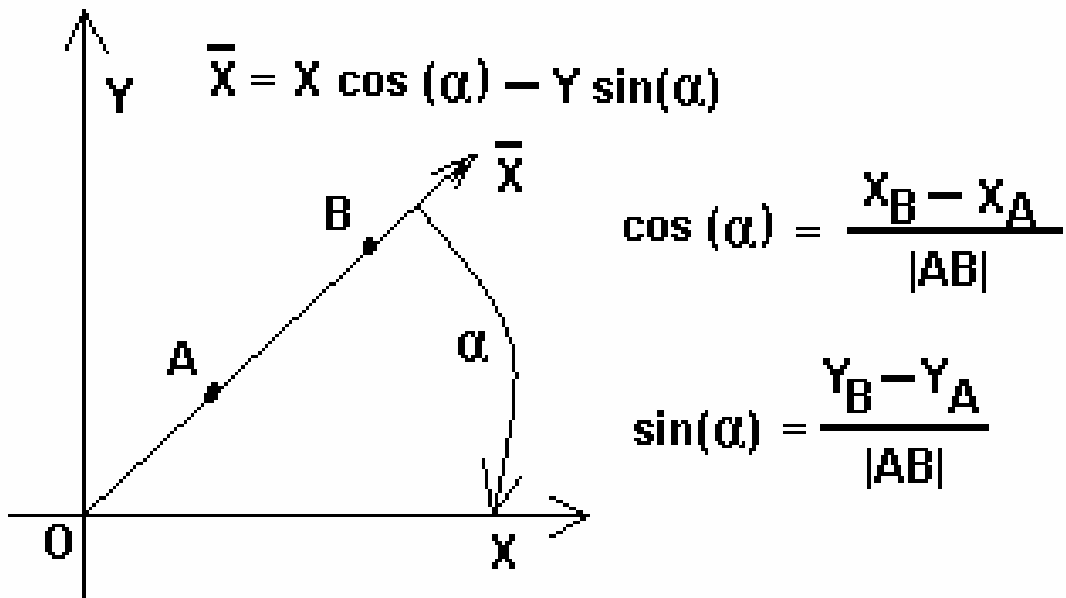
20)

21)

Vign ( .47)

$X$

$OX$  ( .60).



.60.

```

Vign(X,V,P,D) :=
  return 0 if  $|X^{(1)} - X^{(2)}| < 10^{-9}$ 
   $h_0 \leftarrow |X^{(0)}|$ 
   $h_1 \leftarrow \frac{X^{(1)} \cdot X^{(2)} - (|X^{(1)}|)^2}{|X^{(1)} - X^{(2)}|}$ 
   $h_2 \leftarrow \frac{(|X^{(2)}|)^2 - X^{(1)} \cdot X^{(2)}}{|X^{(1)} - X^{(2)}|}$ 
  return  $\begin{pmatrix} 2 \cdot \max(|h_1|, |h_2|) \\ 2 \cdot \text{Dv}(0.5, h_1, h_2) \\ 2 \cdot |h_0| \end{pmatrix} \cdot \frac{1}{\sqrt{1 - (|P|)^2}}$  if  $V = 0$ 

```

Далее по тексту программы (рис.50)

.61.

Vign

9.2.

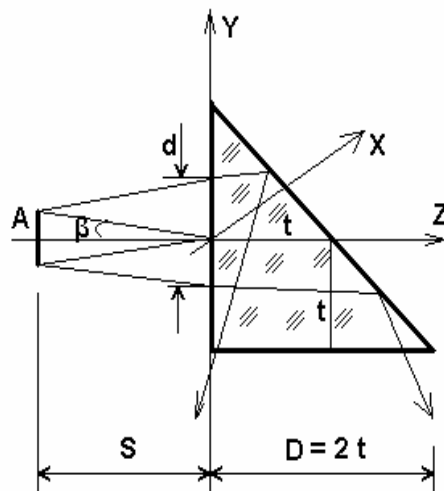
( )

$\beta = \pm 5^\circ$  (.62).

20  
d = 10

0%

D.



.62.

$|c| < 0.1$

$$\begin{aligned}
M &:= 20 & j &:= 0..M-1 & c &:= 0.1 \\
\alpha_1 &:= \text{runif}\left(M, \frac{-c}{60} \cdot \text{deg}, \frac{c}{60} \cdot \text{deg}\right) & \psi_1 &:= \text{runif}(M, 0, 2\pi) \\
\alpha_2 &:= \text{runif}\left(M, \frac{-c}{60} \cdot \text{deg}, \frac{c}{60} \cdot \text{deg}\right) & \psi_2 &:= \text{runif}(M, 0, 2\pi) \\
\alpha_3 &:= \text{runif}\left(M, \frac{-c}{60} \cdot \text{deg}, \frac{c}{60} \cdot \text{deg}\right) & \psi_3 &:= \text{runif}(M, 0, 2\pi) \\
\text{SystR}_j &:= \begin{pmatrix} \mathbf{s}' & \begin{matrix} -30 & -40 & \frac{-100}{3} & 5 & \mathbf{d} \end{matrix} \\ \mathbf{t} & \begin{matrix} 0 & \boxed{10} & \boxed{10} & 0 & \mathbf{z}_v \end{matrix} \\ \mathbf{p+i q} & \begin{matrix} 0 & \frac{-\sqrt{2}}{2}i & 0 & -20 & \mathbf{z}_0 \end{matrix} \\ \mathbf{D} & \begin{matrix} 20+i & 20+i & 20+i & 0 & \mathbf{v} \end{matrix} \\ \Delta \alpha & \begin{matrix} \alpha_{1j} \cdot e^{i \cdot \psi_{1j}} & \alpha_{2j} \cdot e^{i \cdot \psi_{2j}} & \alpha_{3j} \cdot e^{i \cdot \psi_{3j}} & 5 \cdot \text{deg} & \mathbf{\beta} \end{matrix} \\ \Delta \rho & \begin{matrix} 0 & 0 & 0 & \frac{\pi}{2} & \mathbf{\Psi} \end{matrix} \end{pmatrix} \\
& \qquad \qquad \qquad \text{SystR}_j
\end{aligned}$$

Pupil.

$$S' = \frac{n'}{n} S.$$

(80)

SystR<sub>0</sub>.

$$D := \text{ModelMir}(\text{SystR}_0)_1 \quad D = \begin{pmatrix} (3,1) \\ (3,1) \\ (3,1) \\ 0 \end{pmatrix} \quad D_0 = \begin{pmatrix} 10 \\ 5 \\ 10 \end{pmatrix} \quad D_1 = \begin{pmatrix} 14.5 \\ 6.667 \\ 13.334 \end{pmatrix} \quad D_2 = \begin{pmatrix} 18.999 \\ 8.333 \\ 16.667 \end{pmatrix}$$

*D*

ModelMir,

*D*<sub>2</sub>.

*t*

*D*<sub>2</sub>

$$: 18,999 < 2t=2*10.$$

# MolelMir

$x_j \quad y_j, j=0,1,\dots,19. \text{ C}$

stdev

( )

$A_j := \text{ModelMir}(\text{SystR}_j)_0$

$x_j := (A_j)_0$

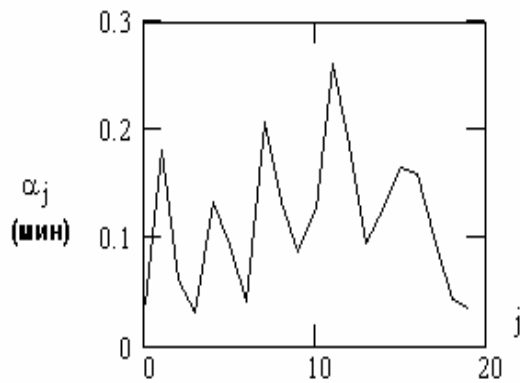
$\text{stdev}(x) = 8.692 \times 10^{-4}$

$y_j := (A_j)_1$

$\text{stdev}(y) = 9.282 \times 10^{-4}$

$$\alpha_j := \text{atan} \left[ \frac{\sqrt{(x_j - \text{mean}(x))^2 + (y_j - \text{mean}(y))^2}}{|(\text{SystR}_0)_{0,2}|} \right] \cdot \frac{60}{\text{deg}}$$

↑  
Последний отрезок



$$K = \frac{\text{mean}(\alpha)}{c} = 1.15$$

10.

10.1.

)

d,

$n_2,$

$n_1$ .

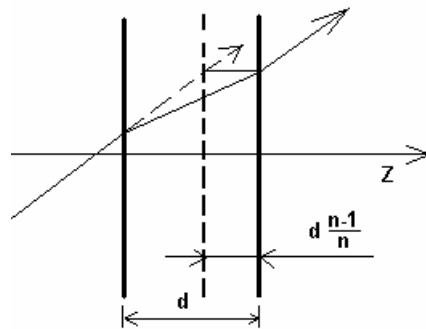
$$\begin{aligned} \frac{n_2}{S'_1} - \frac{n_1}{S_1} &= 0, & S'_1 &= \frac{n_2}{n_1} S_1, \\ S_2 = S'_1 - d, & \rightarrow S_2 = \frac{n_2}{n_1} S_1 - d, \\ \frac{n_1}{S'_2} - \frac{n_2}{S_2} &= 0. & S'_2 = \frac{n_1}{n_2} S_2 = S_1 - \frac{n_1}{n_2} d & \rightarrow \Delta S = d + S'_2 - S_1 = \frac{n_2 - n_1}{n_2} d \end{aligned}$$

(81)

$(n_1=1)$

$$\Delta S = \frac{n-1}{n} d. \tag{82}$$

(.63):



.63.

10.2.

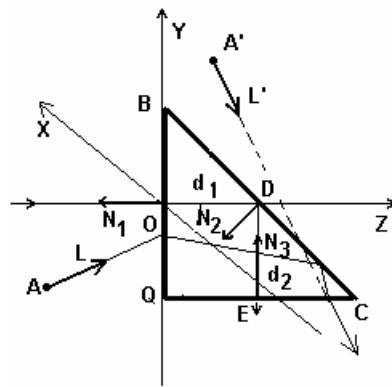
, [5])

“ ” ( ..

: , -

OZ

( .64).



.64.

$d_1, d_2$  . .

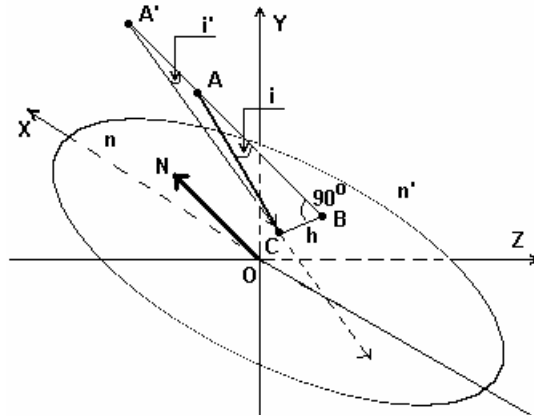
$L$ .

$$: L = -N.$$



$n \quad n'$

(.65).



.65.

$N -$ ,  $i -$ ,  $i' -$ ,  $h$   
 $A'C$ ,  $AC$ ,  $OBC$ ,  $AB$   
 $A$ ,  $A'$ ,  $C$ ,  $B$ ,  $O$

$A(x_0, y_0, z_0) /$   
 $n \cdot AC = L(p_0, q_0, m_0).$   
 $\vec{N}(p, q, m).$

$A'(x', y', z') /$   
 $n' \cdot A'C = L'(p', q', m').$   
 $S = BA,$   $S' = BA'.$   
 $h,$

$S' = \frac{S}{n \cdot \cos(i)} \sqrt{n'^2 - n^2 \sin^2(i)}. \quad (83)$

$S' = \frac{\text{sign}(n')S}{n \cdot \cos(i)} \sqrt{n'^2 - n^2 \sin^2(i)} \quad (84)$

$$A', \quad (84)$$

$$A' = B + \frac{A-B}{S} S' = B + (A-B) \frac{\text{sign}(n')}{n \cdot \cos(i)} \sqrt{n'^2 - n^2 \sin^2(i)}. \quad (85)$$

[6],

$$B(x_1, y_1, z_1),$$

$$B = I_0 \cdot A, \quad I_0 = - \begin{matrix} p^2 - 1 & p \cdot q & m \cdot p \\ p \cdot q & q^2 - 1 & m \cdot q \\ p \cdot m & q \cdot m & m^2 - 1 \end{matrix}, \quad (86)$$

$$\cos(i) = - \frac{(N, L)}{n}. \quad (87)$$

(85), (86) (87), :

$$A' = M \cdot A, \quad M = [I_0 - (I_0 - E)t], \quad t = \frac{\text{sign}(n')}{(N, L)} \sqrt{n'^2 - n^2 + n^2 \cdot (N \cdot L)^2}. \quad (88)$$

$$t = -1$$

$$M = 2I_0 - E. \quad (88)$$

$$(88) \quad (86)$$

$N$

[7].

$$\mu \cdot p = p_0 - p' = -\Delta p$$

$$\mu \cdot q = q_0 - q' = -\Delta q, \quad (89)$$

$$\mu \cdot m = m_0 - m' = -\Delta m$$

$\mu -$

(89):

$$\Delta p = \frac{p}{m} \Delta m, \quad \Delta q = \frac{q}{m} \Delta m. \quad (90)$$

$$p_0^2 + q_0^2 + m_0^2 = n^2,$$

$$p'^2 + q'^2 + m'^2 = n'^2, \quad p \quad q \quad (90)$$

$$\frac{\Delta m}{m} :$$

$$\frac{\Delta m^2}{m^2} + 2(p_0 p + q_0 q + m_0 m) \frac{\Delta m}{m} - (n'^2 - n^2) = 0. \quad (91)$$

$$\mu = -\frac{\Delta m}{m} = \tau - \text{sign}(n' \cdot \tau) \sqrt{\tau^2 + n'^2 - n^2}, \quad \tau = (L \cdot N) = (p_0 p + q_0 q + m_0 m) \quad (92)$$

$$\begin{pmatrix} p' \\ q' \\ m' \end{pmatrix} = \begin{pmatrix} p_0 & p \\ q_0 & q \\ m_0 & m \end{pmatrix} \left[ (N \cdot L) - \text{sign}(n' \cdot (N \cdot L)) \right] \sqrt{(N \cdot L)^2 + n'^2 - n^2}. \quad (93)$$

$$C = I \cdot A, \quad I = \frac{1}{(N \cdot L)} \begin{pmatrix} -(mm_0 + qq_0) & p_0 \cdot q & m \cdot p_0 \\ p \cdot q_0 & -(pp_0 + mm_0) & m \cdot q_0 \\ p \cdot m_0 & q \cdot m_0 & -(pp_0 + qq_0) \end{pmatrix}. \quad (94)$$

$$\begin{pmatrix} p'_0 \\ q'_0 \\ m'_0 \end{pmatrix} = \frac{1}{n'} \begin{pmatrix} 0 & p \\ 0 & q \\ n & m \end{pmatrix} \left[ nm - \text{sign}(n' \cdot nm) \right] \sqrt{(nm)^2 + n'^2 - n^2}. \quad (95)$$

$$\begin{pmatrix} p'_0 \\ q'_0 \\ m'_0 \end{pmatrix} = \begin{pmatrix} p & 0 \\ 2m \cdot q & 0 \\ m & -1 \end{pmatrix} \quad (95)$$

$(p, q, m)$   $(p'_0, q'_0, m'_0)$  (95)  $OZ$ .

$OZ$

$OXY$   $OY$ ,

$OX$

$$\cos(\gamma) = m, \quad \sin(\gamma) = \sqrt{1-m^2}, \quad \sin(\varphi) = \frac{p}{\sqrt{1-m^2}}, \quad \cos(\varphi) = \frac{q}{\sqrt{1-m^2}},$$

$$R = \begin{pmatrix} 1 & 0 & 0 & \cos(\varphi) & \sin(\varphi) & 0 \\ 0 & \cos(\gamma) & \sin(\gamma) & -\sin(\varphi) & \cos(\varphi) & 0 \\ 0 & -\sin(\gamma) & \cos(\gamma) & 0 & 0 & 1 \end{pmatrix} =$$

$$\begin{pmatrix} \cos(\varphi) & \sin(\varphi) & 0 & \frac{q}{\sqrt{1-m^2}} & \frac{p}{\sqrt{1-m^2}} & 0 \\ -\cos(\gamma)\sin(\varphi) & \cos(\gamma)\cos(\varphi) & \sin(\gamma) & -\frac{m \cdot p}{\sqrt{1-m^2}} & \frac{m \cdot q}{\sqrt{1-m^2}} & \sqrt{1-m^2} \\ \sin(\gamma)\sin(\varphi) & -\sin(\gamma)\cos(\varphi) & \cos(\gamma) & \frac{p}{q} & \frac{q}{m} & m \end{pmatrix} \quad (96)$$

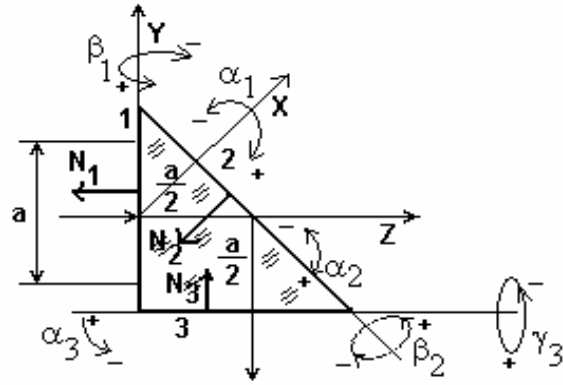
$$p=q=0, \quad R=E, \quad -$$

$$(-90^0),$$

$$(.66).$$

$$N_1(0, 0, -1), \quad N_2 \left( 0, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right), \quad N_3(0, 0, -1).$$

$$: L(0, 0, 1). \quad n=1, n'=1,5.$$



.66.

[8]

[9]

$$\Delta\alpha_1 = \Delta\beta_1 = \Delta\alpha_2 = \Delta\beta_2 = \Delta\alpha_3 = \Delta\gamma_3 = 2'$$

,  $N=20$ .

MathCAD :

$$\alpha_1 = \text{runif}(N, -\Delta\alpha_1, \Delta\alpha_1), \quad \alpha_2 = \text{runif}(N, -\Delta\alpha_2, \Delta\alpha_2), \dots$$

$$\overset{\rho}{N}_1 \left( \sin(-\beta_1), \quad \sin(\alpha_1), \quad -\sqrt{1 - \sin^2(\beta_1) - \sin^2(\alpha_1)} \right)$$

$$\overset{\rho}{N}_2 \left( \sin(-\beta_2), \quad \sin\left(-\frac{\pi}{4} + \alpha_2\right), \quad -\sqrt{1 - \sin^2(\beta_2) - \sin^2\left(\alpha_2 - \frac{\pi}{4}\right)} \right),$$

$$\overset{\rho}{N}_3 \left( \sin(-\gamma_3), \quad \sin(\alpha_3), \quad -\sqrt{1 - \sin^2(\gamma_3) - \sin^2(\alpha_3)} \right)$$

$$\text{Syst} = \begin{pmatrix} p_0 & p_1 & p_2 & p_3 \\ q_0 & q_1 & q_2 & q_3 \\ n & m_1 & m_2 + i & m_3 \end{pmatrix}$$

.67.

$$\text{React}(L, N, n, n') := \left[ \begin{array}{l} NL \leftarrow N \cdot L \\ \left[ L - N \cdot \left( NL - \text{sign}(n' \cdot NL) \cdot \sqrt{NL^2 + n'^2 - n^2} \right) \right] \end{array} \right]$$

$$\text{PrizmP}(S) := \left[ \begin{array}{l} k \leftarrow \text{cols}(S) - 1 \\ L \leftarrow S^{(0)} \\ n \leftarrow L_2 \\ L_2 \leftarrow \sqrt{1 - (L_0)^2 - (L_1)^2} \\ \text{for } i \in 1..k \\ \quad \left[ \begin{array}{l} n_t \leftarrow \text{if}(i = 1, 1, n) \\ n'_t \leftarrow \text{if}(i = 1, n, \text{if}(i = k, 1, -n)) \\ L \leftarrow \text{React}(L, S^{(i)}, n_t, n'_t) \end{array} \right. \\ L \end{array} \right]$$

.67.

React, -

(95), L -

, N -

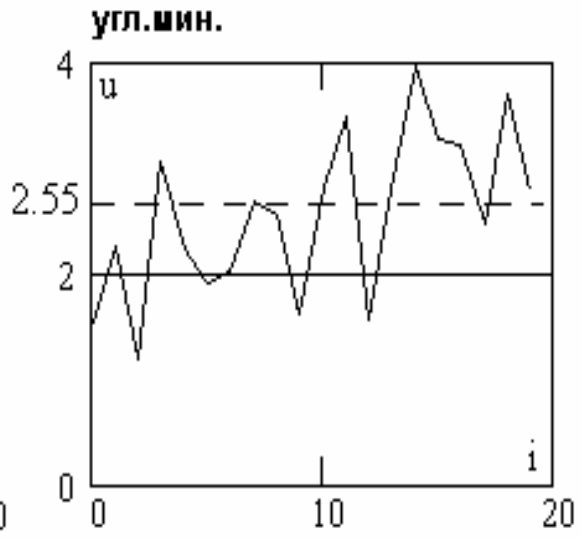
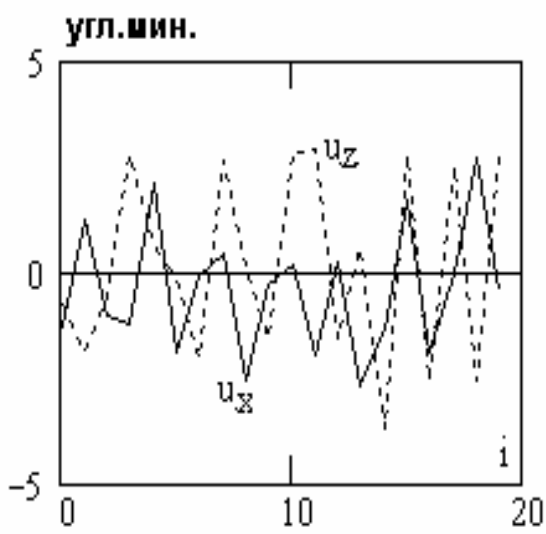
, n, n`

(k),

(L),  
(n).

$(n_b, n'_i)$

(.68)



.68.

( )

( )

2,55

OXY.

(96).

.69.

( ),

$(p, q, m)$ ,

( React)  
(R).

```


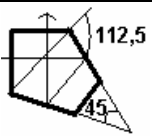

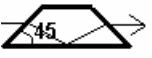
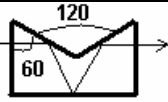
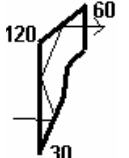
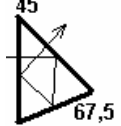
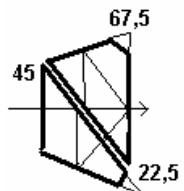
PrizmR(S) :=
  k ← cols(S) - 1
  L ← S(0)
  n ← (L)2
  (L)2 ← √(1 - [(L)0]2 - [(L)1]2)
  P ← (0 0 1)T
  for i ∈ 1..k
    nt ← if(i = 1, 1, n)
    n't ← if(i = 1, n, if(i = k, 1, -n))
    P ← React(P, S(i), nt, n't)
    L ← React(L, S(i), nt, n't)
    p ← P0 / n't
    q ← P1 / n't
    m ← P2 / n't
    R ← ⎛
      ⎜
      ⎝
       $\frac{q}{\sqrt{1-m^2}}$    $\frac{p}{\sqrt{1-m^2}}$   0
       $\frac{-m \cdot p}{\sqrt{1-m^2}}$    $\frac{m \cdot q}{\sqrt{1-m^2}}$    $\sqrt{1-m^2}$ 
      p  -q  m
      ⎞
      ⎟
      ⎠ if |m| < 1 - 10-9
    R ← ⎛
      ⎜
      ⎝
      1 0 0
      0 1 0
      0 0 1
      ⎞ otherwise
    L ← R.L
  L

```

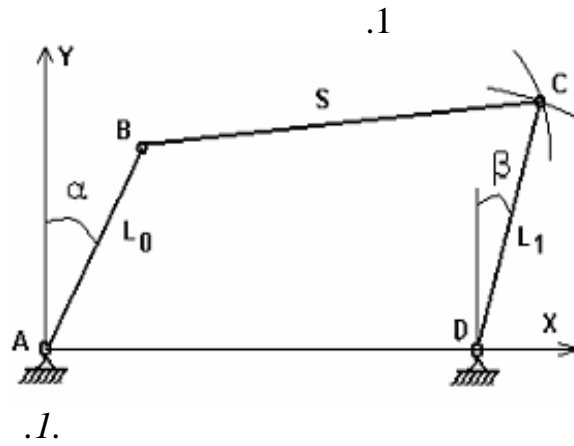
.69.



5.

1		1
2		$2 + \sqrt{2}$
3		2
4		3,337
5		$3\sqrt{3}$
6		$2,5\sqrt{3}$
7		$1 + \frac{\sqrt{2}}{2}$
8		$\frac{5 + 3\sqrt{2}}{2}$

1



AB, DC, BC. : = A D, [1]

$$A(0,0), |AB|=L_0, |BC|=S, D(x_1, y_1), |CD|=L_1.$$

$$x_B = -L_0 \sin(\alpha), y_B = L_0 \cos(\alpha), \quad (1)$$

$$B D, S L_1 ( .1)$$

$$\begin{aligned} (x_C - x_B)^2 + (y_C - y_B)^2 &= S^2 \\ (x_C - x_1)^2 + (y_C - y_1)^2 &= L_1^2 \end{aligned} \quad (2)$$

$$y_C = a - bx_C, \quad a = \frac{S^2 - L_0^2 - L_1^2 + x_1^2 + y_1^2}{2(y_1 - y_B)}, \quad b = \frac{x_1 - x_B}{y_1 - y_B}. \quad (3)$$

$$Ax_C^2 - 2Bx_C + C = 0, \quad (4)$$

$$A = 1 + b^2, \quad B = x_B + b(a - y_B), \quad C = x_B^2 + (a - y_B)^2 - S^2.$$

$$x_C = \frac{B + \sqrt{B^2 - AC}}{A}. \quad (5)$$

DC

$$0, \quad |x_1 - x_C| < \varepsilon,$$

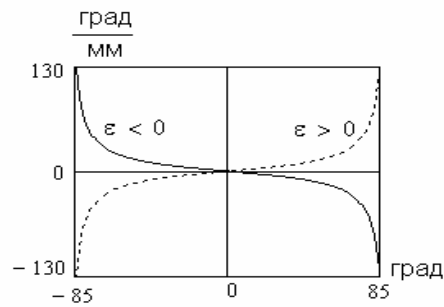
$$\beta = \arctg \frac{y_1 - y_C}{x_1 - x_C} + 90^\circ, \quad \beta < 90^\circ, \quad (6)$$

— ,  $10^{-9}$ .

2.6).

$$\beta_{L_0}(\alpha) = \frac{F(\alpha, x_1, y_1, L_0 + \Delta L, L_1, S) - \alpha}{\Delta L}. \quad (7)$$

$$(7) \quad x_1 = S = 10, y_1 = 0, L_1 = L_2, \Delta L = 10^{-3}$$



.2.

0.

$k = 0,2.$

$F(\alpha, x_1, y_1, L_0, L_1, S) :=$	$\alpha \leftarrow \alpha \cdot \text{deg}$ $x_B \leftarrow -L_0 \cdot \sin(\alpha)$ $y_B \leftarrow L_0 \cdot \cos(\alpha)$ $a \leftarrow \frac{S^2 - L_1^2 - L_0^2 + x_1^2 + y_1^2}{2 \cdot (y_1 - y_B)}$ $b \leftarrow \frac{x_1 - x_B}{y_1 - y_B}$ $A \leftarrow 1 + b^2$ $B \leftarrow x_B + b \cdot (a - y_B)$ $C \leftarrow x_B^2 + (a - y_B)^2 - S^2$ $D \leftarrow B^2 - A \cdot C$ $x_c \leftarrow \left( \frac{B + \sqrt{D}}{A} \right)$ $y_c \leftarrow a - b \cdot x_c$ $t \leftarrow 0$ if $ x_1 - x_c  < 10^{-9}$ $t \leftarrow \text{atan} \left( \frac{y_1 - y_c}{x_1 - x_c} \right) \cdot \frac{1}{\text{deg}} + 90$ otherwise $t \leftarrow t - 180$ if $t > 90$ $t$
---------------------------------------	---

.3.

- 1)
- 2)
- 3)
- 4)
- 5)

MathCAD

	1	2	3	4	5	6	7	8	9
	$x_0$	$y_0$	$r_0$	$x_1$	$y_1$	$r_1$	$L_0$	$L_1$	$S$

2

- 1) 6
- 2) Kardinal.
- 3)
- 4) MathCAD.
- 5) [3] ( 1)

- 6)
- 7)
- 8)

1.

$f'=200,0$ $D=30$	$r$	$n_c$	$t$
	144,66		
		1,6213	3,0
	54,44		
		1,5139	9,0
	-189,13		

2.

2

1	$r_1, r_2$	8	$r_2, n_1$	15	$r_3, t_2$
2	$r_1, r_3$	9	$r_2, n_2$	16	$n_1, n_2$
3	$r_1, n_1$	10	$r_2, t_1$	17	$n_1, t_1$
4	$r_1, n_2$	11	$r_2, t_2$	18	$n_1, t_2$
5	$r_1, t_1$	12	$r_3, n_1$	19	$n_2, t_1$
6	$r_1, t_2$	13	$r_3, n_2$	20	$n_2, t_2$
7	$r_2, r_3$	14	$r_3, t_1$	21	$t_1, t_2$

3

1) 7

2) Pupil.

3)

4)  $A_k, (k=1,2,3)$

5)  $y$ ,

$x_k$ .

$$: \delta x_k = \frac{\Delta y}{A\sqrt{n}},$$

$n -$

6) 4

( . .2)

7)

$y$ .

4

8

[4],

1-4.

1) -1 ,

2)

3)

4) -54

1.

“ -1 ”

( $f=32,59$ ,  $s=-765 - -2966$  ,  $=646,1$  ,  $2 =65^\circ$ )

	$R( )$	$t( )$	$n$		$D( )$
1	12,078				14,2
		$1,95 \pm 0,02$	1,746046	9	
	18,707				13,08
2		$0,06 \pm 0,02$	1		
	13,459				12,52
		$1,95 \pm 0,1$	1,746046	9	
	81,66				11,84
		0	1		
	81,66				11,84
		$0,78 \pm 0,1$	1,746231	4	
3	19,999				10,26
		$1,76 \pm 0,01$	1		
	-48,98				9,16
		$0,78 \pm 0,02$	1,746231	4	
4	11,092				8,56
		$1,17 \pm 0,01$	1		
	24,55				8,66
		$1,76 \pm 0,02$	1,746231	4	
5	-24,55				8,60
		0,9	1		
	$\infty$				8.29
			1		



2. “ ”  
 $(f' = 34,807$  ,  $s = \infty$  ,  $= 0,65628$  ,  $2 = 56^\circ)$

	$R( )$	$t( )$	$n$		D
1	14,093				15,82
		1,078	1,739667	19	
	15,065				12,98
		0,04	1		
2	12,745				15,52
		1,212	1,739667	19	
	14,970				12,98
		0,03	1		
3	12,815				14,15
		1,950	1,739667	19	
	34,081				10,88
		0,8	1		
4	-118,557				13,88
		1,557	1,666602	2	
	9,855				10,74
		1,94	1		
5	28,977				13,54
		3,510	1,739667	19	
	-12,827				10,21
		0	1		
6	-12,827				10,21
		0,593	1,512184	14	
	-270,594				9,47
7		0,9	1		
	$\infty$				12,43

3.

“ ”

 $(f=34,80, s=\infty, = 0,65628, 2 = 63^\circ)$ 

	$R(\ )$	$t(\ )$	$N$		$D$
1	12,474				14,91
		1,69	1,739667	19	
	16,493				14,08
		$0,29 \pm 0,02$	1		
2	12,706				13,30
		2,18	1,739667	19	
	28,71				12,20
		$0,8 \pm 0,01$	1		
3	-130,32				12,20
		1,07	1,666602	2	
	9,954				10,05
		$1,98 \pm 0,02$	1		
4	32,28				10,00
		3,02	1,739667	19	
	-13,243				9,60
		0	1		
5	-13,243				9,53
		0,62	1,512184	14	
	-146,22				9,40
6		0,9	1		
	$\infty$				12,429

4.

“ -57”

 $(f=34,94, s = \infty, = 0,65628, 2 = 63^\circ)$ 

	$R(\ )$	$t(\ )$	$N$		$D$
1	9,772				11,5
		$3,81 \pm 0,01$	1,656004	3	
	18,621				9,42
		$0,98 \pm 0,01$	1		
2	-29,850				9,42
		$0,94 \pm 0,01$	1,658782	28	
	10,471				8,3
		$0,81 \pm 0,01$	1		
3	20,51				8,45
		$2,55 \pm 0,02$	1,738053	9	
	-20,51				8,5
		0,9	1		
	$\infty$				9,98

5

9

[4]:

- 1)
- 2)
- 3)
- 4) -54

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